

A Survey on the Tandem Queueing Models

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Abstract: Queueing systems play a pivotal role in modeling and optimizing service processes across diverse industries. Among various Queueing models, the tandem Queueing model stands out as a fundamental and versatile framework. Understanding and effectively analyzing these systems is crucial for improving efficiency, minimizing delays, and enhancing customer satisfaction. This survey reviews the historical development of tandem Queueing theory, elucidates core concepts, and highlights its significance in modern service systems. We also provide an overview of analytical tools and techniques used to evaluate tandem Queueing system performance, covering both classical and contemporary methodologies. Purpose of this paper is to provide an in-depth evaluation and comprehensive overview of tandem Queueing models.

Keywords: Vacations, Tandem queue, Breakdown, Blocking, Reneging, Retrial, and Priority.

Introduction

The theory of Queueing is a mathematical discipline that examines the behavior of wait times and associated systems. It has diverse applications in telecommunications, computer networks, manufacturing, and transportation. A fundamental concept within this field is the tandem Queueing model. Tandem Queueing systems are a crucial building block for analyzing and optimizing complex, multi-stage service systems, where components are arranged in series.

The tandem Queueing model describes a sequence of connected queues or servers, where the output of one queue becomes the input of the next, effectively capturing the dynamics of real-world systems with multiple service stations. Understanding and

managing such systems is crucial for enhancing service quality, minimizing delays, and optimizing resource utilization. This survey provides a comprehensive overview of the tandem Queueing model, exploring its theoretical foundations, practical applications, and analytical techniques for performance evaluation. By examining tandem queue dynamics, this work offers valuable insights for researchers, practitioners, and decision-makers seeking to leverage Queueing theory to increase system efficiency and customer satisfaction.

Tandem Queueing analysis has significant implications for various domains. In computer networks and telecommunications, it contributes to network design and management, ensuring desired quality of service, particularly in delay-sensitive and throughput-critical applications. Similarly, in service-oriented systems, understanding tandem Queueing dynamics is crucial for meeting service level agreements and designing systems that fulfill contractual obligations. Tandem queues often arise in situations with sequential resource sharing, and analysis provides insights into optimal resource allocation across stages to achieve overall system goals. A comprehensive survey would benefit researchers and practitioners, serving as a valuable resource summarizing existing knowledge, methodologies, and areas requiring further research.

This paper is organized as follows: Section 2 summarizes Queueing theory models, while Section 3 explores tandem Queueing models. Subsequent sections examine specific tandem Queueing scenarios, including bulk services (Section 4), servers taking vacations (Section 5), blocking (Section 6), impatient customers (Section 7), server breakdowns (Section 8), retrial queues (Section 9), and priority tandem queues (Section 10). Finally, Section 11 concludes the paper.

2. Queueing system

A Queueing system, also known as a queue or waiting line system, is a mathematical model that analyzes the behavior of entities (e.g., customers, data packets, or tasks) as they arrive at a service facility, wait in line if necessary, and receive service. Queueing systems have diverse applications across various domains. Figure 1 illustrates a tandem Queueing system, providing a classical representation of this model.

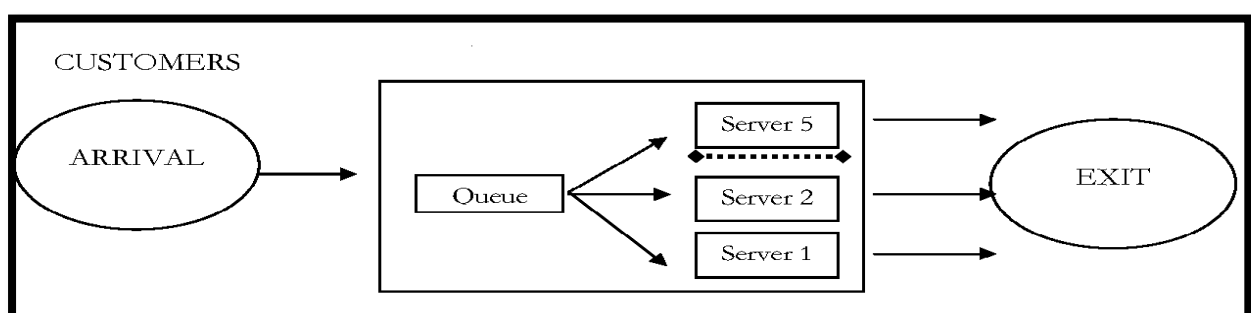


Figure1: Tandem Queueing system

In the early twentieth century, Danish engineer Agner Krarup Erlang pioneered Queueing theory while employed at the Copenhagen Telephone Company. Tasked with addressing call congestion in telephone networks, Erlang recognized that calls arrived stochastically and required processing by a limited number of operators. His groundbreaking work [1] aimed to understand and optimize switchboard capacity and efficiency, leading to the development of Queueing hypotheses that would become foundational to the field.

Meisling's [2] pioneering work laid the foundation for bridging the gap between discrete-time and continuous-time Queueing systems, offering a valuable framework for analyzing and understanding Queueing behavior in various real-world applications. Neuts' [3] gives analysis of Matrix analytic methods in Queueing Theory was driven by the quest for algorithmic techniques and yielded results that are well-suited for practical computer implementations, facilitating the application of Queueing theory in real-world scenarios. Fomundam's [4] comprehensive survey serves as a valuable resource for healthcare professionals and researchers, offering insights into the diverse applications of Queueing theory in healthcare and aiding in the development of strategies to enhance healthcare service delivery. Vilaplana's [5] study on a Model of Queueing theory for cloud computing, the utilization of an open Jackson network offered a systematic framework for assessing and fine-tuning cloud service response times, ultimately enhancing the overall efficiency and reliability of cloud-based applications. Lluís Mas's [6] study on a Queueing theory model for fog computing offers a practical framework for identifying and addressing performance challenges, ultimately assisting in the efficient design and operation of fog computing systems.

Ali Hamzah-Najim's [7] OPNET 14.5 Modeler was utilized for evaluating Queueing theory in Wi-Fi applications. Highlights the significance of choosing the right Queueing mechanism to ensure efficient data handling and improved user experiences in wireless networks, shedding light on the trade-offs between different organizational methods. Aliyu and Diocou [8] explored the potential of an analytical Queueing model, which utilizes Software-Defined Networking (SDN), to manage IOT traffic within 5G networks. By incorporating Markov's Queueing model alongside SDN controllers and Network Function Virtualization (NFV), their work provides a comprehensive framework for intelligent and efficient traffic routing, facilitating effective communication across various 5G access networks and their respective endpoints through the deployment of virtual switches. Recently Xu and Liu [9] analyzed two stage tandem Queueing system with priority and clearing service in the

second stage, In this paper the stationary distribution of the system is derived using the matrix analytic method.

3. Tandem Queueing model

A tandem Queueing model is a type of Queueing system used in Queueing theory and operations research to analyze and understand the behavior of entities as they pass through a sequence of interconnected queues or service stages. In a tandem Queueing model, entities move from one queue to the next in a sequential fashion, often representing a series of processing or service stages that an entity must undergo before reaching its final destination. Tandem Queueing models find application in various fields, including telecommunications, manufacturing, healthcare, and logistics where understanding the flow of entities through multiple interconnected stages is essential for optimizing system performance and resource allocation. Analyzing tandem Queueing systems allows researchers and practitioners to gain insights into how entities progress through a sequence of services and help identify potential areas for improvement or optimization within the system. A Schematic representation of the tandem Queueing model is given in Figure 2.

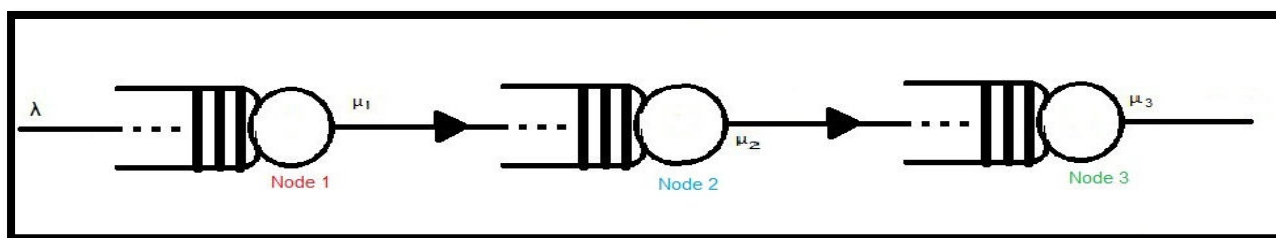


Figure2: Tandem Queueing Model

Henry D. Friedman's [10] study on Tandem Queueing System Efficiency addressed the intricate challenges of tandem Queueing systems, including issues related to waiting time, queue length, and occupancy time, presenting novel reduction methods that continue to inform the analysis and optimization of these systems. Kan Wu et al [11] study on Examining Tandem Queues with Limited Buffer Capacity delves into the dynamics of Queueing systems with limited buffer capacity, revealing that service rates become dependent on arrival rates in such scenarios. Through the innovative application of virtual interruption, the research yields crucial insights by providing estimates for the average waiting time within double tandem queues, advancing our understanding of Queueing behavior under finite buffer constraints.

Chesoong Kim and Sergey Dudin's [12] investigation into the Admission Control Methods in Priority Tandem Queueing Models introduced a novel numerical method for accurately determining the stationary distribution of the Queueing system. Their research also extended to the calculation of loss probabilities at both tiers of the

tandem, offering valuable insights into admission control strategies and the overall performance of this Queueing model. Valentina Klimenok et al. [13] study centered on a tandem Queueing system with correlated input and cross-traffic dynamics is used. Within this research, they introduced a groundbreaking recursive technique tailored to accurately compute the stationary distributions and loss probabilities, thus providing valuable insights into the behavior and management of such intricate tandem Queueing configurations.

Exploring Delay Tandem Queueing Model Analysis using Mobile Queues investigated by Ahmad Al Hanbali [14]. This study reveals that the queue length approximation demonstrates impressive performance, particularly in scenarios characterized by low traffic loads. Amuthan's [15] study on Congestion-Adaptive Routing Using the Dynamic Multi-Stage Tandem Queue Simulation in MANET, delves into the estimation of average thresholds, employing dynamic multi-stage tandem queue modeling as an inspiration drawn from stochastic probability. Larionov's [16] study Using NS-3 Simulation to calibrate a Tandem Queueing Model with PH Service Time in a Multihop Wireless Network, the research focuses on employing the moment matching method within the model. This method is utilized to extract Phase-Type (PH) distributions based on sampled data obtained during packet transmission within the context of an IEEE 802.11 channel.

4. Tandem queue with bulk services

A tandem queue with bulk services refers to a Queueing system that consists of multiple interconnected queues arranged sequentially, where each queue provides service to arriving entities that are served in groups or batches, rather than individually. Tandem queues with bulk services have garnered significant attention from researchers due to their relevance in modeling and optimizing various real-world systems, ranging from production lines to computer networks. Tandem queues with bulk services have been studied by numerous researchers.

G.V. Krishna Reddy [17] investigated a Tandem Queue in Unit III with Three Multi-Server Modules and Bulk Service. Within this model, a modified geometrical matrices method was applied to derive the static frequency vector representing the customer count in the queue. Furthermore, the study confirmed the fulfillment of the stability criterion. Xiuli Chao and Michael Pinedo [18] conducted a study on Examining Arrivals in Tandem Queues with No Intermediate Buffers. In this model, they calculated the estimated system time using an expression and comprehensively investigated the various features and properties of this expression.

Mishev and Haghighi [19] conducted the Three-Stage Stochastic Recruiting Model, This can be thought of as a Tandem Queueing Process with Bulk Arrivals and Erlang Phase-

Type Selection, indicated as $M^x/M(n,n)/1 - M^y/Er/1$. Within this model, they employed decomposition techniques to ascertain the generating functions and mean number of applications in the initial two phases of the system. Sumita and Masuda[20] researched on Tandem Queues, Infinite Servers, and Correlated Service Times. Within this model, they employed a set of renewal equations to calculate the binomial moments associated with the joint occupancy distribution, providing insights into the behavior of such complex Queueing systems.

Aparajitha and Srinivasa Rao [21] unveiled the Two-Node Tandem Waiting Model with Direct Arrivals to Both Service Stations, Includes State and Time-Dependent Phase-Type Service. This novel framework has demonstrated exceptional utility in the management and monitoring of communication systems, spanning LAN, WAN, and MAN environments. Vishnevsky [22] conducted research titled State Reduction in Tandem Queueing Devices with Correlated Arrivals Analysis. This investigation highlights that while various techniques are suitable for smaller tandems; larger tandems necessitate more extensive efforts to determine appropriate analysis methods.

Harrison [23]exploredthe Distribution of Timing of Response in a Tandem Queue Combined with Batch Processing. Within this study, a recursion formula was devised by differentiating the transform at the origin, facilitating the efficient computation of arbitrary moments of the LaplaceResponse time through iterative methods. Recently Dudin et al [24] presented the influence of the thresholds on the system performance measures and analyzed tandem queues with Multi-server and group service at the second stage. In this model, Customer arrivals are described by the versatile Markov arrival process. The system is studied via consideration of a multi-dimensional continuous-time Markov chain.

5. Server vacation in tandem queue

A dual queue with a server vacation is a Queueing system that consists of multiple sequential service stations (queues), where servers are allowed to go on vacation or take breaks when there are no customers in their respective queues. This vacation policy is often introduced to save on operational costs when there is insufficient workload to keep all servers busy continuously. When customers arrive at a queue with a server on vacation, they may either wait for the server to return from vacation or join a separate queue for the next available server in the tandem. This modeling approach is useful for analyzing systems with variable workloads and dynamic server availability, such as call centers and customer service operations.

Krishna Reddy et al. [25] conducted a study on TandemQueueing along with General Batch Service and System Vacations. This research utilized a modified matrix

geometric technique to construct vectors of stationary probability for the total number of customers that have to wait and be served in the first unit, as well as the total number of consumers in the reserve, for both models. The study also presents numerical results to support the findings. Padmaja Shashi [26] conducted research on Server Vacation and Tandem Queueing Systems. The primary focus of this investigation was to identify the appropriate batch size that optimizes system time for the customer and to derive the system's average number of customers, employing the standard birth-death technique.

Gupta and Kavusturucu [27] conducted research on Machine Vacations in Tandem Manufacturing Systems. In their paper, they utilized T-tests to assess disparities between the two outcomes. The developed methodology was determined to be robust and highly accurate. Sinu Lal et al [28] Examined Tandem Queues with Numerous Servers and a Customized Server performing a Vacation Approach. In this model, they defined the system's stability criteria and successfully derived the constant probability distribution using the matrix analytic method. T.S. Sinu Lal et al [29] investigated a Tandem Queue with a Customized Server Using a Vacation Strategy. Within this model, they established a stability requirement for the entire system and established the stationary distribution using matrix analytic methods. Additionally, the study computed various performance indicators to assess system performance.

6. Tandem queue with blocking

Tandem queues with blocking refer to a class of Queueing systems where multiple service stations or queues are arranged sequentially, and if a customer cannot be served at one station due to full capacity, they are blocked or delayed until they can proceed to the next available station in the tandem. Blocking occurs when there is no space available in a queue for incoming customers, which can lead to delays and affect the overall system's performance. Modeling and analyzing such tandem queues with blocking are essential for understanding and optimizing various real-world processes, such as manufacturing, telecommunications, and computer networks, where customers or tasks move through a series of stages, and capacity limitations at each stage can lead to congestion and delays. IN figure 3 represents the Tandem queue with Blocked Customers.

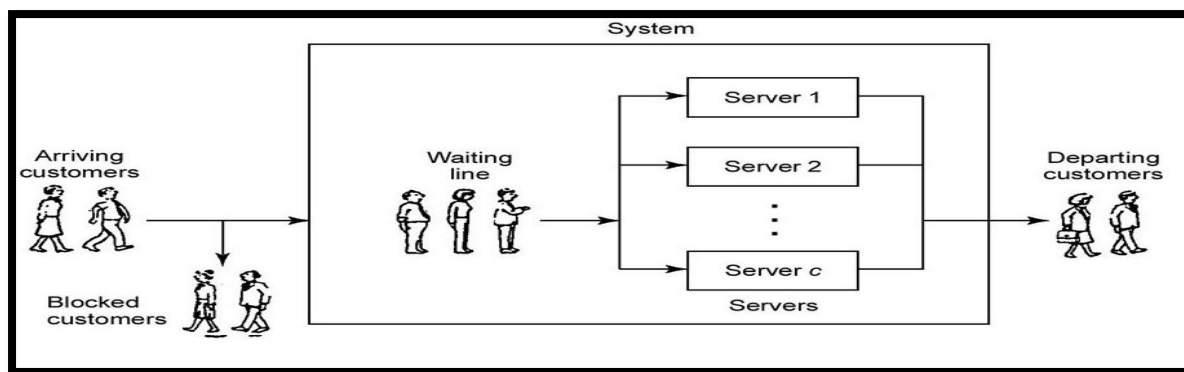


Figure 3: Tandem Queueing System with Blocked Customers.

Perros and Foster [30] conducted an experiment named Queue System Analysis Blocking Processes. In this research, the authors developed both exact and approximate constraints for mean blocking times in various network configurations. Furthermore, they derived stability criteria for two-stage networks. Liu [31] explored the throughput Properties of Tandem Utilizing Queueing Networks with Blocking based on Decomposition. This study employed the sample path method to examine a fundamental attribute of throughput within Networks of tandem processing with blocking. The research establishes a connection between outgoing decomposition strategies and the throughput bounds.

Grassmann and Drekić [32] conducted research on Tandem Queues with Blocking: Analytical Solutions. The purpose of this model was to determine the two lines' combined distribution in equilibrium. The authors utilized generalized Eigenvalues as a key tool to compute this joint distribution. Seo et al [33] investigated whether Waiting moments are constant in M-Node Tandem Queues with Manufacturing Blocking. Within this model, various properties were employed to address a buffer distribution problem with constant waiting time probabilistic assumptions. Haghghi [34] conducted research on Tandem Queueing Devices Featuring Task Division, Feedback, and Blocking. In this study, techniques were employed to calculate various performance metrics, including the distribution of queue lengths as an entire at steady state, queue length durations, mean waiting times, and the station and system probabilities activity statuses (busy or unbusy). Recently, Chandra Sekar Reddy et al. (35) conducted a steady-state analysis of tandem queues with blocking and feedback, employing the spectral expansion method. Earlier, Huard and Alfa (35) examined response times in tandem queues with blocking, Markovian arrivals, and phase-type services. Their approach involved constructing a Markov chain based on the age of the leading customer in the first queue.

Zhchinski et al [36] conducted research on Modeling, Analysis, and Operation Insights from Tandem Queues with Time Variation and Blocking Using Fluid Models

with Reflection. In this study, the author introduced time-varying fluid models for tandem networks with blocking, offering valuable insights into modeling, analysis, and operational aspects.

Shin et.al [37] delves into Tandem Queues with General Blocking Nodes: An Approximate Analysis. Within this research, a decomposition-based approach for system approximation is presented, with a numerical assessment of its effectiveness. Recently Palpandi [38] presented a new approach for the fuzzy two-stage tandem queue with blocking based on the fuzzy structured Element (FSE) theory.

7. Tandem queue with impatient customers

Reneging in a tandem queue refers to a phenomenon in Queueing theory where customers or entities waiting in a sequence of multiple queues (tandem queues) decide to leave the queue before they are served. This decision to abandon the queue typically occurs due to factors such as long waiting times, impatience, or dissatisfaction with the service. Reneging can have serious consequences for the operation and effectiveness of a Queueingsystem, as it affects metrics like customer waiting times, queue lengths, and overall service quality. Analyzing and regulating renegeing behavior is an important aspect of the design and optimization of Queueingsystems. Figure 4 represents the tandem queue with Balking and Reneging.

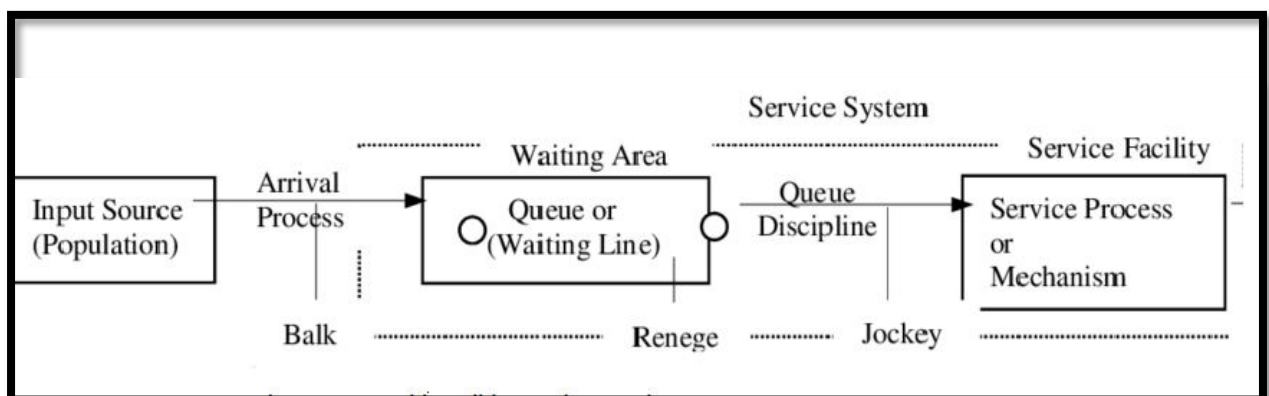


Figure 4: Tandem Queueing System with Balking and Reneging.

Chandra Shekhar et.al [39] conducted a study on Infinite Capacity Tandem Queueing Networks and Reneging. This investigation primarily centered on the estimation of the queue size's stationary distribution within the system. Azadeh et al. [40] researched modeling and optimizing complex tandem Queueing systems with server failures and reneging, subject to budget constraints. They proposed a simulation-optimization framework for managing infinite capacity tandem queues with failing servers and reneging. Similarly, Wang et al. (41) analyzed a Markovian two-station tandem queueing network with impatient customers, simplifying the analysis by combining the recursive renewal reward theorem with Queueing and Markov chain decomposition (QMCD).

8. Tandem queue with server failure or breakdown

Each service station in the tandem queue may encounter breakdowns or failures. When a station breaks down, it becomes temporarily unavailable for serving customers until it is repaired or restored to working condition. Many research projects are carried out in tandem with queues with server breakdowns due to their work with real-time applications.

Evaluation of Dual Tandem Queues with Finite Buffer Capacity and Non-Overlapping Service Times during a Break was conducted by Wu and Zhao [42]. Within this research, they derived approximate models utilizing Virtual interruptions and priority queues. Nadarajan and Jayaraman [43] conducted a study of serial Queues with General Bulk service, Random Failure, and Vacations. Within this research, they utilized geometric matrix theory to determine the steady-state probabilities and constraints.

Dimitriou et.al [44] explored the Investigation of a Tandem Queue with Coupled Processors Susceptible to Global Failures. Within this model, a method was introduced to construct the stationary joint queue length distribution generating function for both network states, utilizing a solution in the form of the boundary value problem of Riemann-Hilbert. Tsai et.al [45] presented a Performance Evaluation of Open Queueing Networks that is vulnerable to breakdowns and repairs. This model focuses on the study of scheduling techniques and sensitivity analysis within the system.

Yi Peng et.al [46] conducted research on Stochastic Queueing System Based on Levy with Server failures and vacations investigated. Within this model, the study derived the limiting distribution of the workload process by leveraging Levy process properties and employing the Kella-Whitt martingale approach. Jayaraman, Natarajan & Sitrarasu [47] researched the Analysis of a Bulk Service General Queue with a Server Failure Arrival Rate. Within the framework of this model, to compute the stationary probability vector for the queue's customer count and create the stability requirement, a matrix geometric algorithmic approach was used. Agrawal and Jain [48] conducted a study titled Optimal Strategy for Bulk Queueing Systems with a Variety of Server Failures. In this research, they utilized the geometric matrix approach to compute the average length of the queue and various system statistics. Recently Dudin et al [49] analyzed Queueing system with server breakdowns & Individual customer abandonment. This model is provided steady-state probability for GI/M/1 type with a finite state space.

9. Connection between Tandem Queue and Retrial Queue.

A retrial queue is a Queueing system where customers who are unable to enter a busy server or queue immediately due to congestion or unavailability may reattempt to enter the system after a certain period. Avrachenkov and Yechiali [50] delved into Tandem Optimization Queue Blocking with Joint Recurrence Queues. Within this study, two optimization problems were explored, focusing on reducing the entire amount of time a job spends in the system, determining the optimal order of queues, and allocating specific capacities to the primary queues.

Kim et.al [51] investigated Tandem Retrial Waiting Techniques with Arrival Flow Correlation and Second Station Process Described by a Markovian Chain. This work comprehensively addressed ergodicity requirements and provided methods for calculating steady-state probabilities within the system. Klimenok and Dudina [52] conducted research on the Analysis of Retrial Tandem Queues with a Controllable Repeated Attempts Strategy. In this study, they focused on computing the steady-state system distribution content with embedded time intervals at any given period.

10. Tandem queue with priority

In a tandem queue with priority, the server prioritizes high-priority units over low-priority ones, as described by Madan, K.C. [53]. This priority system follows two principles: Non-pre-emptive priority service and Pre-emptive priority service. Chesoong Kim et al [54] studied the Priority tandem queueing system with retrials and reservation of channels as a model of the call center. Sasikala et al [55] studied the study of serial Queues with General Bulk Service Random failure and Vacations. They derived the probability-generating function of queue length from an arbitrary time epoch. Recently Xu and Liu studied [56] analysis of a two-stage tandem Queueing system with priority and Clearing service in the second stage. In this paper, the stationary distribution of the system is derived using the matrix analytic method and spectral expansion technique.

11. Conclusion

Tandem Queueing models continue to be at the forefront of Queueing theory, facilitating a deeper optimization of complex processes remains as vital as ever. Future research in this area promises to further refine these models and their applications, ultimately leading to more resilient and responsive systems across diverse domains. This article discusses how we have conducted a comprehensive review of tandem Queueing models, focusing on specific phenomena and characteristics associated with the behavior of queues in tandem configurations understanding real-world systems, and driving improvements in operational efficiency. As technology evolves and new challenges emerge, the role of tandem Queueing models in modeling and

Researchers, engineers, computer and network systems, manufacturing systems, and telecommunications systems can all benefit from these models.

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