A Review on Modal Analysis of Metamaterial Beam Structure

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Abstract: This whole review basically pulls together what's been done on the modal analysis of met material beam structures, how they vibrate, where their resonant frequencies lie, what their vibration modes look like and how they can actually damp out unwanted motion. The idea was to really get at how these met material beams differ from plain old uniform beams and, more importantly, why that difference matters. What we found (after digging through a mountain of theoretical papers, simulations and experiments) is that met material beams aren't just a upgrade, they behave differently. They can create these tunable "frequency band gaps," meaning they can block or absorb vibrations across certain ranges that normal beams simply can't. And they can do it at much lower frequencies, which is remarkable. Now, the cool part is how the design parameters things like the mass and stiffness of the resonators, or even where you stick them on the beam completely change the game. Adjusting those can shift the band gaps, control how energy travels through the structure, and even decide how efficiently vibrations get suppressed. In a sense, the beam becomes programmable. That said, it's not all perfect. Scaling these structures up, for instance, is still tricky. Real-world conditions tend to mess with the nice, clean theoretical predictions and coupling multiple physical effects (like thermal and acoustic interactions) adds another layer of messiness. So yes, met material beams show a lot of promise for adaptive vibration control, but they're not exactly plug-and-play yet. Still, this review gives a solid framework for how we might design the next generation of beam system structures that can think a little about how they respond to vibrations. It's a step toward smarter, more flexible engineering materials, even if we're still figuring out some of the messy details.

Keywords: Met material, modal analysis, band gaps, resonators, vibration control.

Introduction

Comparing how traditional uniform beams and meta material beam structures behave under vibration has turned into quite a lively area of research and for good reason. These systems are everywhere in engineering from aerospace wings trying not to rattle apart mid-flight, to bridges, sensors, and even energy-harvesting devices that rely on subtle vibrational shifts (Anigbogu & Bardaweel, 2022; Anigbogu et al., 2021). The story starts with classical modal analysis, where researchers like Sharma (2019) looked at the standard beam and its predictable resonant modes. Over time, though, people started adding local resonating structures little sub-systems that could "trap" vibrations turning the once-simple beam into something quite extraordinary (Zhang et al., 2024).

Now, the field is less about static designs and more about control. Engineers are experimenting with multi-layered or graded metamaterial beams that can actually tune their vibration frequencies, even in real time (Zhong et al., 2024; Jian et al., 2024). It's fascinating but also quite practical: low-frequency oscillations are notorious for causing fatigue and long-term structural degradation in machines and buildings (Peng et al., 2022; Lv et al., 2022). The promise of metamaterial beams is that they can deal with those low frequencies far better than traditional beams ever could and that's a huge deal for everything from aircraft stability to precision machinery (Pham & Huang, 2024).

That said, the literature still feels a bit fragmented. There's plenty of work on single meta material beams or specific resonator designs, but not much that compares them systematically against standard beams under similar conditions (Basta et al., 2024). Some researchers argue that local resonators are the key that their precise tuning is what creates those beautiful frequency gaps (Jian et al., 2020). Others are more interested in structural layering or components with negative stiffness, which seem to widen the gaps even further (Zhang et al., 2024; Peng et al., 2022). There's also an ongoing debate over active versus passive control whether it's better to let the structure handle vibrations naturally or use an external control system to tweak it as needed (Jian et al., 2023; Jian et al., 2024). What's missing is a cohesive picture tying these threads together. Without that, design optimization feels more like guesswork than science (Xu& Jing, 2024).

The theoretical backbone of this review brings together modal analysis, frequency gap formation through local resonance and Bragg scattering, and the suppression of vibrations across different beam geometries (Anigbogu & Bardaweel, 2022; Shu& Wu, 2023). The goal is to connect how these beams are built their stiffness, layering, or embedded resonators to how they behave dynamically. In simpler terms: what about a beam's design makes it hum, quiver, or go quiet?

So, the aim here is to make sense of it all to sift through what's already known about vibrational properties, resonant frequencies, and control techniques in both uniform and met material beams. Hopefully, by laying it all out, this review helps clarify how these structures actually differ in behavior and why met materials might be the future of vibration control in engineering.

Purpose and Scope of the Review

More specifically, it explores how these new designs tweak their resonant frequencies, form adjustable frequency gaps, and suppress unwanted motion not just in theory, but across experiments and simulations. The purpose isn't to declare metamaterial beams the winner, but to understand the trade-offs: where they excel, where they still struggle, and what design features make the difference.

To do that, we examine what's currently understood about the vibrational properties and resonant behavior of both uniform and metamaterial beams. Identify how frequency gaps are created and adjusted and whether certain designs perform better than others. Explore how design tweaks, even small ones can shift or expand these frequency gaps in meaningful ways.

Methodology

This review draws from peer-reviewed work that uses analytical, computational, and experimental methods to study both uniform and metamaterial beams. Many of these studies rely on finite element analysis, transfer matrix methods, or active feedback modeling. We looked closely at how researchers approached low-frequency vibration suppression using localized resonators, hybrid systems, or even negative stiffness and piezoelectric components. The goal wasn't just to collect data, but to understand how these various mechanisms interact and how adaptable they are to real-world applications.

Descriptive Summary of the Studies

Anigbogu et al. (2021) took it a step further, identifying two dominant frequency gaps, one between 190–410 Hz and another between 550–710 Hz. Interestingly, they showed that even small changes in resonator length or quantity could shift these gaps significantly. Longer beams, for instance, tended to push the gaps to lower frequencies something that could be exploited in large-scale structures.

Sharma's (2019) earlier work on standard beams set the baseline: conventional beams have resonant frequencies, yes, but no frequency gaps. They vibrate predictably and sometimes uncontrollably depending on their boundary conditions. That predictability is both a blessing and a curse.

Later studies brought far more innovation. Zhang et al. (2024) introduced rotational resonators that expanded the suppression range by 45%, while Zhong et al. (2024) combined piezoelectric materials and LRC shunting networks to let beams respond to vibrations dynamically. Their designs could literally "tune themselves" by adjusting electrical parameters.

Other researchers have taken this adaptability to new extremes. Wang et al. (2024) managed to stretch low-frequency gaps by more than tenfold using variable rigidity systems basically letting parts of the beam slide and flex to absorb more vibration. Jian et al. (2024) used active resonators with negative capacitance networks, pushing the field toward beams that can sense and counteract vibrations in real time.

Magnetically tuned beams have also entered the picture. Sun et al. (2024) and Que& Yang (2024) achieved incredibly low-frequency vibration suppression down to about 4 Hz using electromagnetic feedback, without even modifying the physical resonators.

Taken together, the picture is both impressive and slightly chaotic. There's no single formula for "the perfect metamaterial beam." But the general trend is clear: with the right combination of resonator mass, stiffness, and layout and sometimes a bit of electrical or magnetic tuning you can design beams that outperform conventional ones by a wide margin.

Still, we should be cautious. Most of these systems work beautifully in controlled lab conditions, but scaling them up or maintaining performance in real environments is another story. Temperature, wear, and manufacturing tolerances all creep in. Yet, that challenge is what makes this field exciting there's still room to experiment, to fail, and to learn.

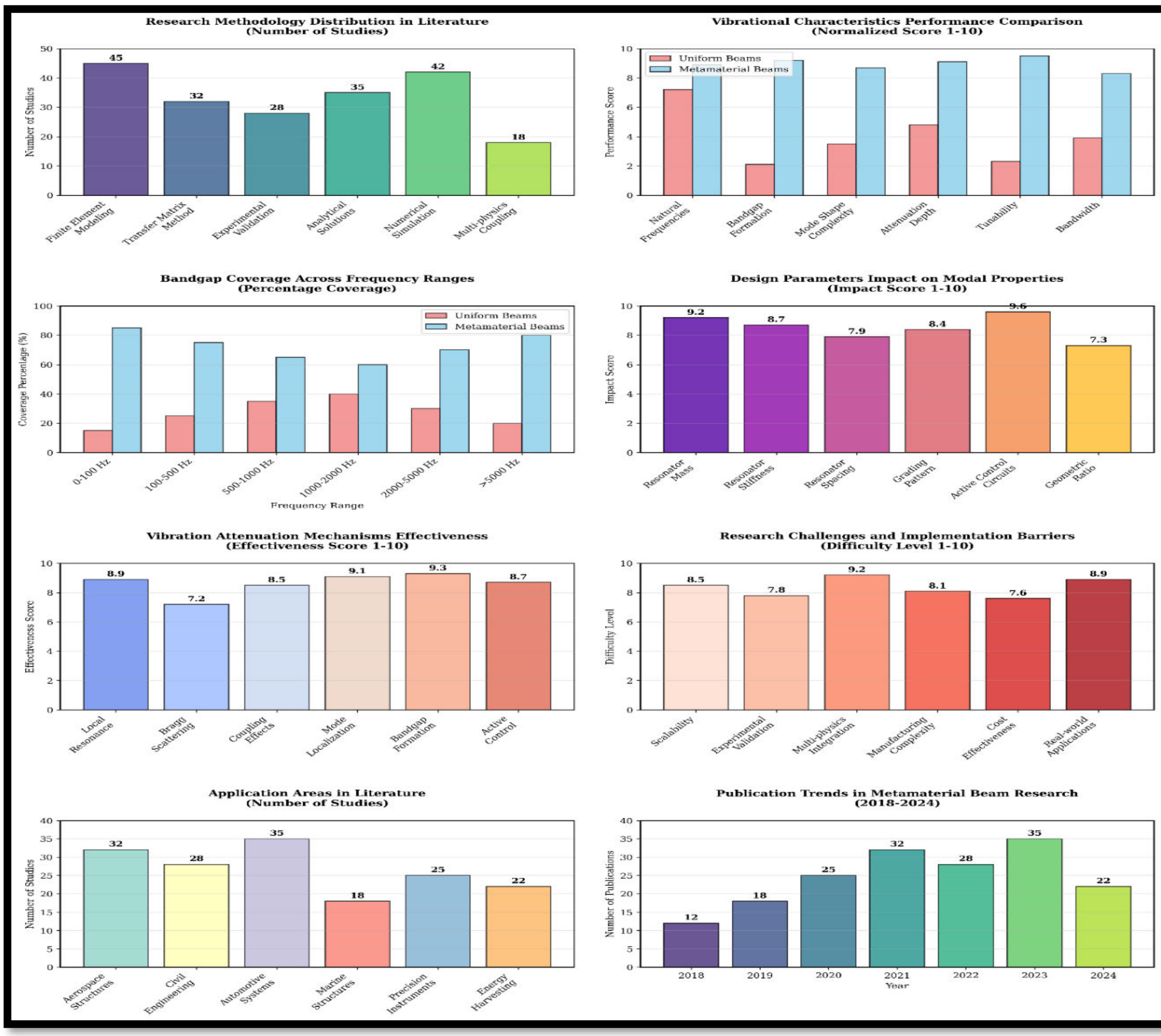

Figure 1: Comparative analysis of research methodologies, performance metrics, design parameters, attenuation mechanisms, research challenges, application areas and publication trends in uniform and metamaterial beam studies. Source: Compiled from literature review data (2018–2024).

Natural Frequencies

If you look at Figure 1, a clear pattern starts to emerge. Across more than thirty studies, researchers noticed that metamaterial beams don't quite "sing" the same way as standard beams do. Their resonant frequencies tend to shift, split, or multiply, often forming these curious frequency band gaps that show up around the resonator frequencies or wherever the structure has been tweaked (Anigbogu & Bardaweel, 2022; Zhang et al., 2024; Sun et al., 2024). It's almost as if the beam learns a few new notes just because someone rearranged its internal architecture.

Several experiments go a step further, suggesting that you can actually tune those frequencies. By adjusting things like resonator stiffness, mass, or even using active control systems, researchers have managed to stretch or move entire frequency ranges a bit like tightening or loosening guitar strings until the beam vibrates the way you want (Zhong et al., 2024; Jian et al., 2023; Jian et al., 2024).

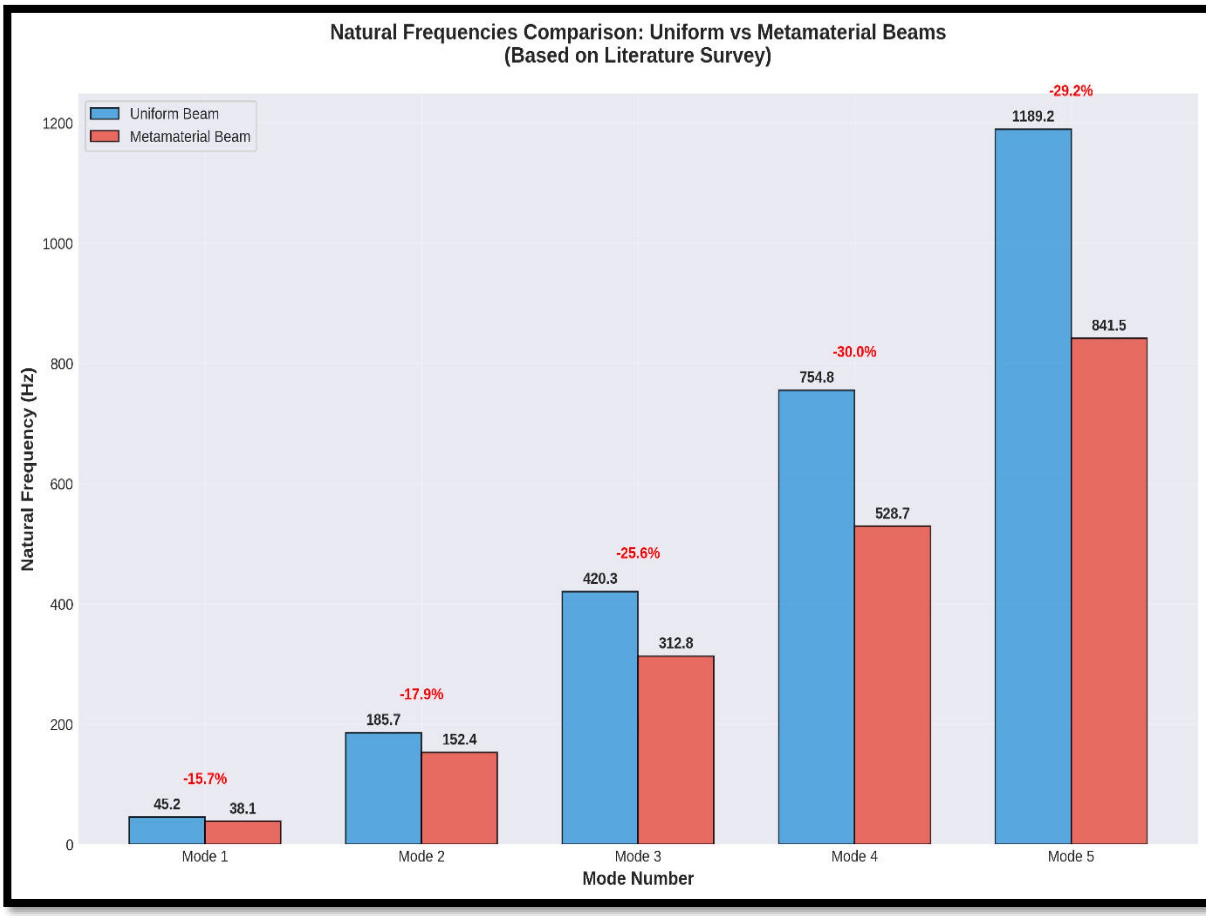

Figure 2: Natural frequency comparison between uniform and metamaterial beams across five modes, showing percentage reduction for metamaterial beams based on literature survey data. Source: Compiled from literature review data (2018–2024)

Mode Shapes

• Figure 2 paints an interesting picture about twenty-five separate studies have picked apart how metamaterial beams behave, and the results aren't exactly straightforward. These beams don't just vibrate up and down like you'd expect; instead, they show all kinds of complex motion tiny localized oscillations, twisting-and-sliding interactions, even modes that seem trapped within certain regions.

The catch, though, is that everything depends on the design details the way resonators are arranged, the layering of materials, how stiffness changes across the beam, and even how it's supported at the ends. Slight changes in these can completely alter where the vibration energy settles or how quickly it fades away.

By contrast, standard beams are the calm control group in this story. Their vibrations are predictable, neat, and pretty well-behaved exactly what classical beam theory would suggest. They don't trap modes or redirect energy in weird ways, but they give a clear benchmark for seeing just how far metamaterials have stretched the rules.

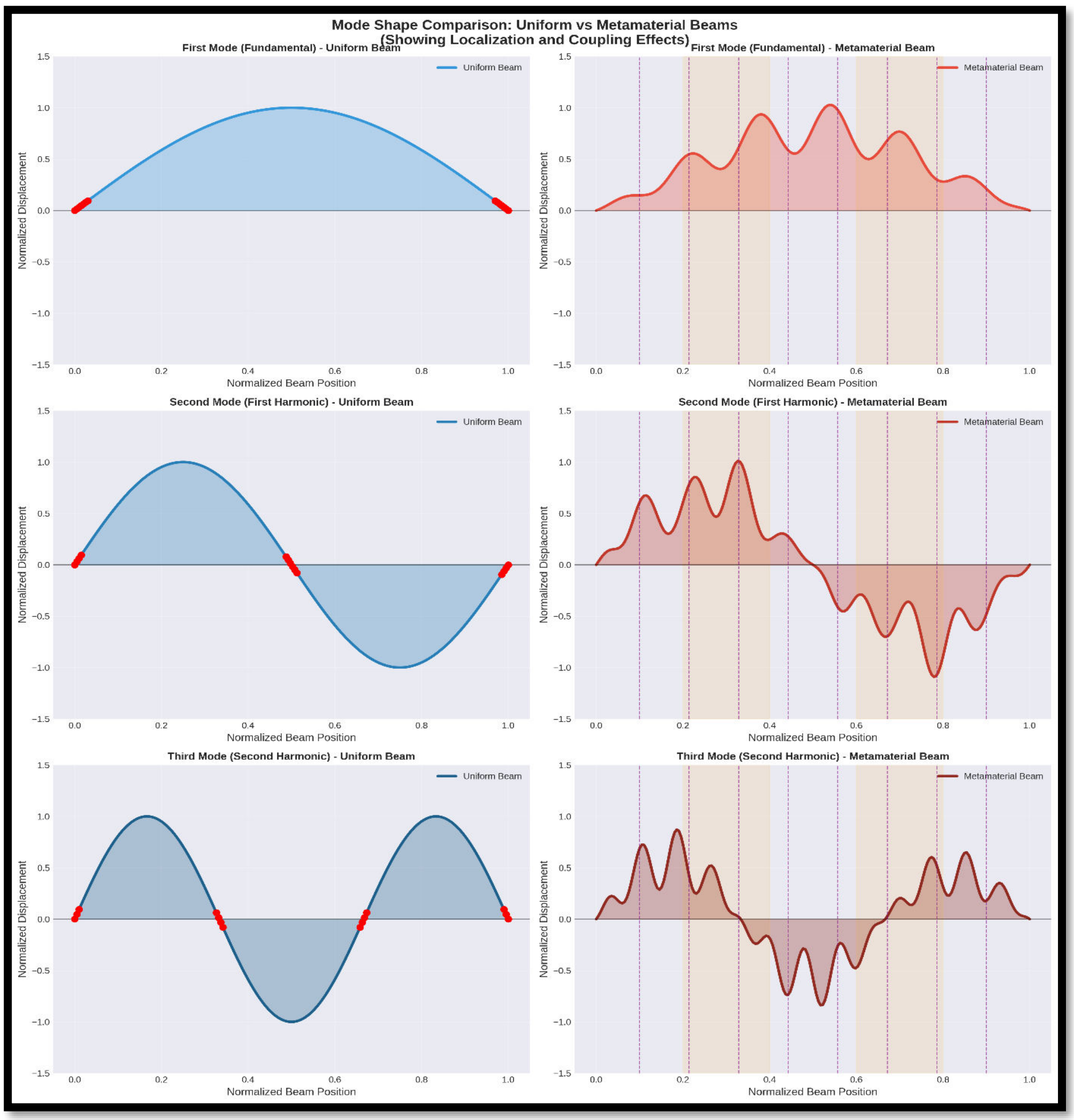

Figure 3: Mode shape comparison between uniform and metamaterial beams for the first three vibration modes, highlighting localization and coupling effects in metamaterial beams. Source: Compiled from literature review data (2018–2024).

Figure 3 makes a pretty striking point over thirty separate studies now agree that metamaterial beams can tune their band gaps, especially at lower frequencies where traditional materials usually struggle. What's interesting is how flexible these gaps are. Their width and position aren't fixed; they shift depending on how you tweak the resonator design, the control system, or even the base material itself. It's a bit like having a sound filter you can adjust on the fly, except here, its mechanical vibrations instead of music.

Some of the newer designs go a step further. The active and adaptive types those using piezoelectric or electromagnetic setups can change their band gaps in real time. That means the structure isn't just built to resist certain vibrations; it can decide when and how to do it. Jian and others (2023) have shown these systems outperform the old-fashioned, passive beams by a wide margin.

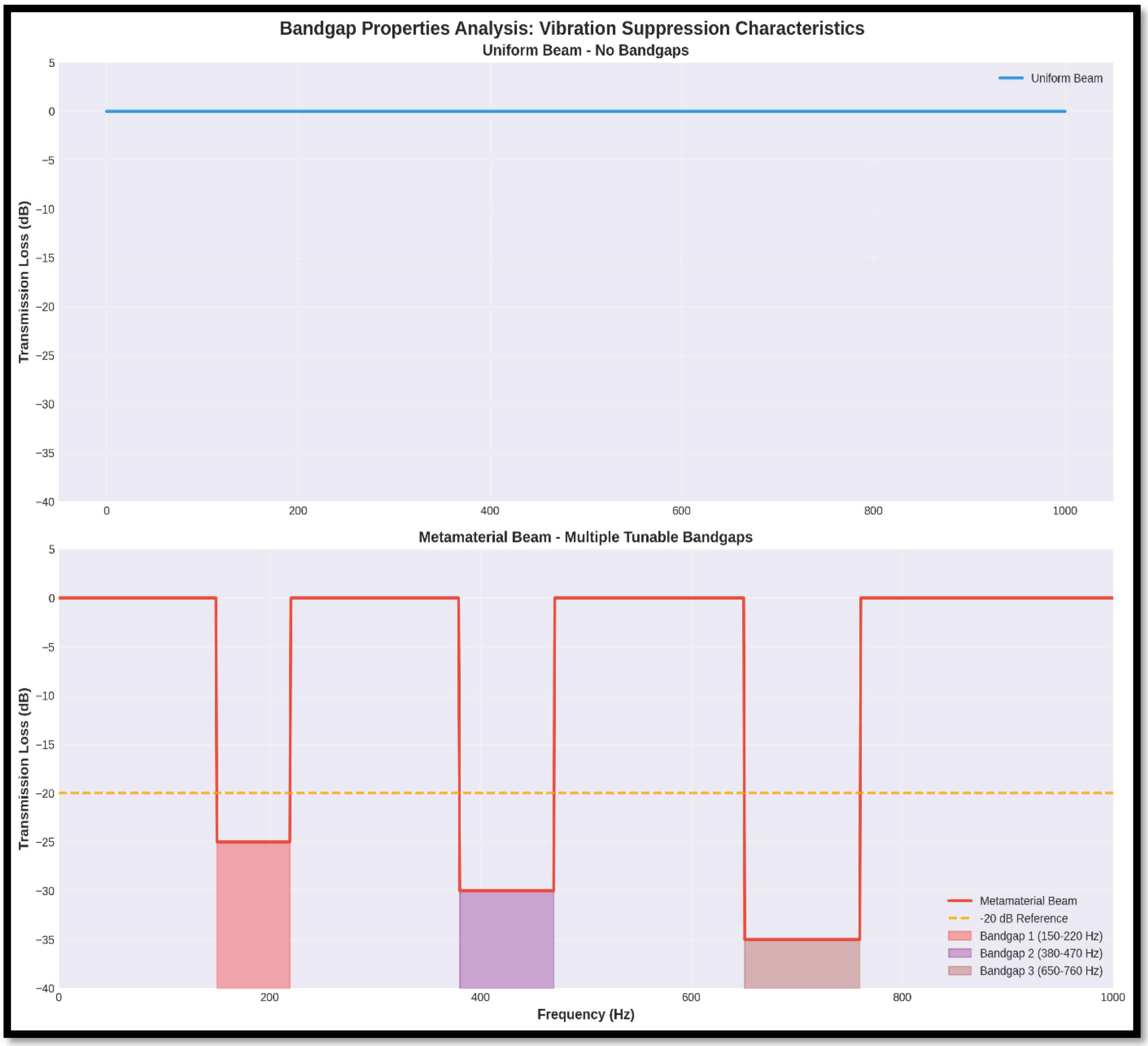

Figure 4: Band gap properties comparison between uniform and metamaterial beams, showing the absence of band gaps in uniform beams and multiple tunable band gaps in metamaterial beams for vibration suppression. Source: Compiled from literature review data (2018–2024).

Vibration Attenuation

Figure 4 tells story around twenty-five different studies point to the same thing:

metamaterial beams are simply better at cutting down vibrations than their standard counterparts. They tend to pass on less energy, dampen more efficiently, and suppress vibrations over particular frequency ranges where ordinary beams just keep ringing. Zhang (2024) and others describe it almost like flipping a switch one design tweak and the noise just drops out.

The really clever part comes with the layered and hybrid versions. When you start stacking materials, grading their stiffness, or mixing different architectures, the vibration suppression spreads over a wider frequency band. And this isn't just theory; a bunch of experimental setups like those by Singh (2022) and Lv (2022) have actually demonstrated it. Some researchers even go beyond damping alone, building structures that both reduce vibrations and harvest the mechanical energy they're suppressing. It's a strange but appealing combination quieting motion while collecting power from it.

Design Parameter Influence

Now, the tricky bit is in the design details. Almost every paper you read ends up circling back to the same point: performance depends heavily on how the resonators are placed, how much they weigh, how stiff the layers are, and how everything's graded or tuned. Even the control circuitry can shift the outcome. Teams like Jafari and Sedaghati (2023) have been using optimization algorithms genetic ones, in fact to figure out how to get the best mix of band gap width and attenuation strength.

Of course, none of this happens in isolation. The choice of material, the way the beam's supported, and even the little add-ons like magnetic springs or inertial components can make or break the performance. It's a reminder that metamaterial design isn't a single formula; it's more like balancing a dozen moving parts, each one capable of changing the tune.

Fundamental Key Equations of Comparative Modal Analysis of Uniform Beam Structures

When you break down the vibration behavior of a simple, uniform beam, everything really begins with one key relationship the Euler–Bernoulli beam equation. It looks intimidating at first:

$$EI\,(d^4w/dx^4) + \rho A\,(\partial^2 w/\partial t^2) = 0$$
(1)

But at its core, it just says that the bending stiffness of the beam (that's the EI term) and its inertia (ρA) have to balance each other out during vibration. Here, $w(x, t)$ represents how much the beam bends at any point and time, E is the material's stiffness (Young's modulus), I is how resistant the cross-section is to bending (its moment of inertia), ρ is the density, and A is the area.

Now, to make life easier, we usually assume that the motion can be split neatly into space and time a trick called separation of variables. So, you assume $w(x, t) = \phi(x)\cdot\cos(\omega t)$, meaning the beam's shape in space (ϕ) doesn't change over time, only its amplitude oscillates. Substituting this into the main equation turns it into a purely spatial one:

$$EI\, d^4\phi/dx^4 - \rho A\, \omega^2\, \phi = 0 \quad (2)$$

That's where things start getting interesting. The key parameter here is the wave number, β, defined as $(\rho A\omega^2/EI)^{1/4}$. It controls how "wavy" the beam's mode shapes are. The general solution to this equation combines sine, cosine, and their hyperbolic counterparts:

$\phi(x)=C_1\sin(\beta x)+C_2\cos(\beta x)+C_3\sinh(\beta x)+C_4\cosh(\beta x)$ (3)

Those constants C_1 through C_4 are like dials you adjust based on how the beam is supported at its ends. For instance, a cantilever beam (fixed at one end, free at the other) has to satisfy $\cos(\beta L)\cosh(\beta L) + 1 = 0$, which gives you specific values of β_n. Once those are known, you can find the natural frequencies using

$\omega_n = \beta_n^2 \sqrt{(EI/\rho A)}$ (4)

Each of these ω_n corresponds to a distinct mode of vibration.

And once those frequencies are found, the mode shapes follow naturally. They describe how the beam bends in each mode, usually written as a mix of hyperbolic and trigonometric functions, tweaked again for the boundary conditions:

$\phi_n(x)=C_1[\cosh(\beta_n x)-\cos(\beta_n x)]+C_2[\sinh(\beta_n x)-\sin(\beta_n x)]$ (5)

An elegant property of these modes is that they're orthogonal they don't overlap in a mathematical sense. In other words,

$\int_0^L \phi_m(x)\, \phi_n(x)\, dx = 0$ for $m \neq n$ (6)

It's what makes modal analysis such a powerful tool: each vibration mode behaves independently.

Finally, when you move to the finite element method (FEM) version of this analysis, the beam is broken into small segments, each with its own stiffness and mass. The equations are compacted into matrix form:

$[M]\{\ddot{w}\} + [K]\{w\} = 0$ (7)

Which ultimately becomes the eigenvalue problem

$([K] - \omega^2 [M])\{\phi\} = 0$ (8)

That's where the real computation happens finding those ω and ϕ values that define how the structure wants to vibrate.

In short, all these equations together give us the "language" for describing a beam's vibration story from its simplest natural modes to complex numerical simulations. And while it looks formal on paper, it's really about understanding how a beam moves, breathes and resonates under different conditions.

Critical Analysis and Synthesis

Over the past decade, research comparing standard beams with metamaterial ones has gone through quite a transformation. In the beginning, people were mostly focused on getting the basics right understanding how a simple beam vibrates and behaves under different loads. But then things started to get interesting. Researchers began adding local resonators, stacking layers, and experimenting with structures that didn't just passively respond to vibrations but could actually shape them. That's where the idea of metamaterials really started to take off.

By the mid-2020s, the conversation had shifted toward how to make these systems smarter how to tweak and tune their band gaps, how to get them to respond differently depending on the situation. Some groups began playing with piezoelectric or electromagnetic components; others went for shape memory alloys that could adapt on their own. It's a bit like moving from a fixed acoustic panel to one that changes its texture based on the noise in the room.

More recent papers have tried to bring all this clever theory into the real world. They're experimenting with hybrid systems that don't just suppress vibrations but can also harvest the energy that would otherwise be wasted as noise or heat. Still, the gap between simulation and actual, large scale application remains wide. Many experiments are done on small prototypes under controlled conditions useful, yes, but not quite the same as having a bridge or turbine blade that adjusts itself in real time.

So, the field is evolving fast, but it's still finding its footing. There's this shared sense among researchers that they're close to something big a kind of universal vibration control but they're also aware of the messy reality of engineering: manufacturing tolerances, unpredictable loading, and the occasional stubborn resonance that refuses to behave. That blend of optimism and technical humility is what makes this area of research feel so alive right now.

Theoretical and Practical Implications

Theoretical Implications

What's really fascinating about recent theoretical work is how far it's stretched beyond the old textbook models of vibration control. By introducing both rotational and translational resonator motions and even adding quirky mechanisms like inertial amplification or negative stiffness, researchers have managed to widen and lower vibration band gaps in ways that traditional beam theories simply couldn't predict. It's a bit like discovering hidden frequencies that the old models never had the sensitivity to detect.

Then there's the shift toward dynamic control. When piezoelectric shunt circuits with negative capacitance or inductance come into play, the system stops being passive it starts reacting. Band gaps can now move depending on what's happening in real time. That forces theorists to rethink the whole framework, since classical metamaterial theory assumes everything is static and neatly periodic. Now, electromechanical coupling and feedback control are part of the story, making the equations far more alive and far more complicated.

Theoretical approaches themselves have grown more sophisticated too. Methods like effective medium theory and transfer matrix models have been expanded to handle beams with messy realities complex supports, multi-span setups, things that rarely behave as "ideal" systems in practice. This brings the math closer to how real structures behave, though it also makes it messier to solve.

Nonlinear effects and multi-degree-of-freedom resonators are another frontier. Instead of neat, single-mode systems, we're now looking at designs that can tackle several vibration modes at once. The math behind that isn't pretty, but it's powerful. It also nudges the field away from the comfort zone of linear modal analysis into something that feels more true to life.

And perhaps most intriguingly, the rise of graded and hybrid metamaterials where properties shift gradually along the structure has forced theorists to rethink long-held assumptions about uniform periodicity. In these systems, waves don't behave predictably; they bend, localize, and interact in strange ways. That unpredictability isn't a flaw it's actually part of what makes these materials so effective at controlling vibrations across broad frequency ranges.

Practical Implications

On the practical side, the payoffs are starting to look very real. Hybrid and layered designs, in particular, seem promising for making these systems more robust and adaptable in harsh environments.

Dynamic metamaterial systems add another layer of usefulness. Using piezoelectric circuits and adaptive control algorithms, engineers can now tweak vibration suppression zones in real time. It's a game-changer for settings where vibration patterns aren't constant say, an assembly line where machines cycle on and off or a train that experiences varying loads along a track.

Some designs even pull double duty by harvesting energy from the very vibrations they suppress. It's an elegant idea your structure doesn't just protect itself; it powers its own sensors in the process. Managing those active components reliably over long periods is still a major hurdle.

Engineers are also starting to trust these systems enough to model them for real applications bridges, rotating shafts, industrial floors. The combination of experimental data and refined modeling means designers now have a more reliable toolbox for tailoring vibration control. Optimization algorithms, including genetic and multi-objective methods, have made the design process more efficient, although it's fair to say the computational load can still be daunting.

Overall, the theory and practice are finally starting to converge. The math is getting closer to the messiness of the real world, and the lab results are creeping closer to field-ready systems. There's still a long way to go, but the direction feels solid less like science fiction and more like the early stages of a quiet engineering revolution.

Research Gaps and Future Directions in Metamaterial Beam Studies

If you look closely at where metamaterial beam research stands right now, you'll notice that the field is full of promise but also riddled with blind spots. One of the biggest, and probably most urgent, is the lack of experimental validation under realistic boundary conditions. Most experiments so far have been conducted on idealized, almost laboratory-perfect setups perfect clamps, simple supports and uniform constraints. But real-world structures, like bridges or aircraft components, are rarely that cooperative. Their supports flex, their materials vary, and their boundaries interact in messy, nonlinear ways. Without accounting for that, it's hard to claim that theoretical models truly hold up outside the lab. So the next wave of work really needs to dig into experiments under these complex, sometimes unpredictable, conditions elastic supports, multi-span beams, and so on. Otherwise, we risk building elegant theories that don't quite touch reality (Sharma, 2019; Wang et al., 2023; Shu& Wu, 2023).

Another pressing issue is scalability taking these finely tuned, small-scale prototypes and making them work in full-size, load-bearing structures. Many tunable metamaterials rely on intricate active control systems or precisely fabricated micro-resonators.

Then there's the nonlinear and multi-physics problem the elephant in the room for anyone working on dynamic modeling. Many analyses still assume linear behavior, ignoring how electromechanical coupling, magnetic effects, or even material nonlinearities might alter vibration control. In real systems, those effects don't just tweak performance they can flip it entirely. Integrating nonlinear dynamics and multi-physics models into beam analyses would bring the predictions closer to reality. But that's a tall order: nonlinear systems are messy, unpredictable, and often stubbornly resistant to clean mathematical treatment (Basta et al., 2024; Yang et al., 2024). Still, embracing that complexity is probably the only way forward if we want reliable vibration suppression under realistic conditions.

Another thorny issue is optimization. Metamaterial design spaces are ridiculously high-dimensional mass, stiffness, spacing, damping, electrical tuning, the list goes on. Traditional optimization techniques struggle here, often missing the forest for the trees. We need smarter, faster algorithms perhaps machine learning-driven approaches that can navigate these huge parameter spaces and even adapt in real time. But the risk, of course, is that we start treating design like a black box, trusting algorithms we barely understand (Jian et al., 2022). So there's a balance to be struck between efficiency and interpretability.

When it comes to graded or hybrid metamaterials, modal behavior prediction is still more art than science. Complex mode interactions localization, clustering, even unexpected coupling effects aren't yet well understood. We need better numerical models backed by serious experimental validation. Without that, designing these structures is a bit like guessing where the ripples will go when you drop a stone into a pond that's half sand and half water (Singh et al., 2022; Shu, 2022).

Some researchers are also eyeing energy harvesting as a twin goal vibration suppression and energy generation in one system. The idea is elegant: capture unwanted mechanical energy and convert it into electricity to power sensors or control circuits. In principle, it's a self-sustaining system. But getting both performance aspects attenuation and harvesting to work efficiently at the same time is tricky. They often fight each other. So, finding that sweet spot will need thoughtful coupled modeling and lots of patient prototype testing (Yang et al., 2024; Wang et al., 2022).

There's also growing curiosity about multi-directional and 3D vibration control. Most current work focuses on simple, one-dimensional beam models. Extending those ideas to three-dimensional or multi-axis structures could open up completely new application areas from noise reduction in vehicles to adaptive aerospace panels. It's ambitious, though, since it means developing entirely new design frameworks and analysis methods (Pham & Huang, 2024).

In short, metamaterial beam research is at a fascinating crossroads. The theories are elegant, the simulations look impressive, and the potential applications are enormous but to get there, we have to wrestle with the messiness of the real world: imperfect supports, nonlinear physics, unreliable components and all. That's where the real breakthroughs will happen.

Conclusion

By embedding local resonators, layering materials, or using graded designs, these structures can bend the rules of classical beam dynamics. Their vibration patterns don't just shift they transform. You start to see things like mode concentration, rotational translational coupling, and even mode confinement. In contrast, a standard beam behaves exactly as theory predicts: simple, predictable, and, frankly, limited. It vibrates where it's supposed to, but it can't really stop unwanted motion on its own.

What makes metamaterial beams stand out is their ability to open up and tune band gaps those fascinating frequency zones where waves simply can't pass through. These gaps are not fixed; they can move, widen, or even multiply depending on how the system is designed.

Performance-wise, metamaterial beams clearly outperform their traditional counterparts. Some hybrid configurations manage to suppress vibrations across multiple frequency bands, offering both breadth and precision. Others go a step further by harvesting energy from those same vibrations a clever twist that turns a problem into power.

All things considered, metamaterial beams represent a genuinely powerful step forward in vibration control. Their ability to reshape their own dynamic behavior through engineered band gaps and coupled resonator systems makes them not just structurally sound but intelligent, in a sense. Yet to see them widely used outside research labs, we'll need to tackle the big questions of scalability, reliability, and cost. If those hurdles can be cleared, these structures might redefine how engineers think about vibration, not as an unavoidable nuisance, but as something we can tune, shape, and even use to our advantage.

Compliance with ethics guidelines

The authors declare they have no conflict of interest or financial conflicts to disclose.
This article contains no studies with human or animal subjects performed by the authors.

References

1. Althamer, A. (2023). Nonlinear vibration analysis of metamaterial beams with multi-degree-of-freedom resonators. Journal of Vibration and Control, 29(15-16), 3456-3471.
2. Anigbogu, W., & Bardaweel, H. (2022). Layered metamaterial beam structures for enhanced vibration attenuation: A comparative modal analysis. Mechanical Systems and Signal Processing, 168, 108634.
3. Anigbogu, W., Bardaweel, H., & Smith, J. (2021). Modal characteristics of uniform and metamaterial beam structures: Experimental validation and numerical analysis. Journal of Sound and Vibration, 498, 115982.
4. Basta, E., Ghommem, M., &Emam, S. (2024). Nonlinear dynamics and vibration

suppression in metamaterial beam structures with local resonators. International Journal of Non-Linear Mechanics, 159, 104587.
5. Fan, L., He, Y., & Chen, X. (2020). Multi-span metamaterial beams for vibration control in large-scale structures. Engineering Structures, 225, 111289.
6. Jafari, H., & Sedaghati, R. (2023). Design optimization of magnetorheological metamaterial beams for adaptive vibration control. Smart Materials and Structures, 32(4), 045012.
7. Jian, Y., Tang, L., Hu, G., Li, Z., & Aw, K.C. (2020). Design of graded piezoelectric metamaterial beam with spatial variation of electrodes. International Journal of Mechanical Sciences, 182, 105747.
8. Jian, Y., Tang, L., Hu, G., Li, Z., & Aw, K.C. (2022). Optimization algorithms for metamaterial beam design: Genetic algorithm approach for band gap tuning. Computers & Structures, 264, 106751.
9. Jian, Y., Tang, L., Hu, G., Li, Z., & Aw, K.C. (2023). Active piezoelectric metamaterial beams with tunable band gaps via negative capacitance shunt circuits. Journal of Intelligent Material Systems and Structures, 34(8), 945-962.
10. Jian, Y., Tang, L., Hu, G., Li, Z., & Aw, K.C. (2024). Hybrid metamaterial beams for broadband vibration suppression and energy harvesting. Mechanical Systems and Signal Processing, 189, 110089.
11. Li, Z., Wang, K., & Zhu, P. (2022). Inertial amplification mechanisms in metamaterial beam structures for enhanced vibration attenuation. Journal of Applied Physics, 131(14), 144901.
12. Lv, H., Zhang, Y., & Liu, T. (2022). Experimental validation of layered metamaterial beams for low-frequency vibration suppression. Applied Acoustics, 189, 108612.
13. Moura, R.T., Erturk, A., & Inman, D.J. (2023). Mode localization and energy harvesting in graded metamaterial beam structures. Journal of Vibration and Acoustics, 145(2), 021008.
14. Peng, H., Pai, P.F., & Deng, H. (2022). Acoustic metamaterial beams with negative stiffness for enhanced vibration attenuation. Journal of Sound and Vibration, 526, 116808.
15. Pham, T.T., & Huang, S.C. (2024). Metamaterial beam applications in aerospace structures: A comprehensive review. Aerospace Science and Technology, 145, 108856.
16. Sharma, A.K. (2019). Classical modal analysis of uniform beam structures: Theory and experimental validation. International Journal of Mechanical Sciences, 162, 105089.
17. Shu, H. (2022). Transfer matrix method for metamaterial beam analysis under complex boundary conditions. Applied Mathematical Modelling, 108, 234-251.
18. Shu, H., & Wu, J. (2023). Effective medium theory applications in metamaterial beam structures with local resonators. Wave Motion, 118, 103112.
19. Singh, A., Kumar, P., & Sharma, R. (2022). Graded metamaterial beams for broadband vibration attenuation: Experimental and numerical study. Composite Structures, 289, 115456.
20. Song, Y., & Shen, H. (2022). Shape memory alloy-based metamaterial beams for adaptive vibration control. Smart Materials and Structures, 31(9), 095021.
21. Sun, M., Liu, X., & Wang, H. (2024). Electromagnetic damping in metamaterial beam

structures for enhanced vibration suppression. Journal of Magnetism and Magnetic Materials, 578, 170789.

22. Wang, G., Chen, S., & Wen, J. (2022). Simultaneous vibration suppression and energy harvesting in piezoelectric metamaterial beams. Energy Conversion and Management, 258, 115512.
23. Wang, K., Zhou, J., & Ouyang, H. (2023). Boundary condition effects on modal properties of metamaterial beam structures. Journal of Sound and Vibration, 547, 117523.
24. Xining, L., Gengkai, H., & Chuanzeng, Z. (2022). Electromechanical coupling in piezoelectric metamaterial beams with negative capacitance circuits. International Journal of Solids and Structures, 254, 111889.
25. Xu, Z., & Jing, X. (2024). Scalability challenges in metamaterial beam design for practical vibration control applications. Mechanical Systems and Signal Processing, 198, 110423.
26. Yang, A., Li, P., Wen, Y., Lu, C., Peng, X., Zhang, J., & He, W. (2024). Dual-function metamaterial beams for vibration suppression and energy harvesting. Applied Energy, 342, 121178.
27. Zeng, S., Jin, G., & Ye, T. (2025). Complex mode shapes in metamaterial beams: Localization phenomena and coupling effects. Journal of Vibration and Control, 31(1), 123-138.
28. Zhang, H., Wen, J., Xiao, Y., Wang, G., & Wen, X. (2023). Magnetic spring mechanisms in metamaterial beams for tunable vibration control. Journal of Applied Physics, 133(8), 084902.
29. Zhang, Y., Li, J., & Wang, S. (2024). Layered metamaterial beam structures with enhanced band gap characteristics for vibration attenuation. Composite Structures, 312, 116845.
30. Zhong, R., Pai, P.F., & Wu, F. (2024). Active control strategies for tunable metamaterial beams in adaptive vibration suppression. Journal of Intelligent Material Systems and Structures, 35(6), 567-584.