A mathematical approach of CKD under Fuzzy environment

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Abstract: Purpose of this paper to introduce new four parameter $\alpha, \beta, \gamma, \delta$related to CKD. Validation and some properties of these parameters are also shown. Extending this parameter into Fuzzified form using Zadeh extension rule and developed a new approach for computing improverate of kidney using these parameter under fuzzy environment
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1. Introduction:

Starting from our many daily life situations up to very complex system; making decision is undoubtedly one of the most necessary activities of human being. It is a logical judgment process to identifying and choosing the right alternatives based on the preferences and values of the decision maker with respect to its criteria. In mathematical point of view, there should be some methodology and algorithm through which one can make a logical and proper decision. Recently, decision making processes have become popular in industries, in different managerial level of the concerned department of many organizations because of their global competitiveness, making good planning and to survive successfully in respective marketplace. Therefore, decision making plays a vital role especially in purchase department for reducing material costs, minimizing production time as well as improving the quality of product or service. Decisions arise when varieties of alternatives are there in front of us associated with different criteria. Decision making process tells how decisions are actually made and how they can be made better or more successfully

Chronic kidney disease is one of the most effected by individual one in recent days all around the world. So it is important to study about CKD in all approaches. Mathematics is one of the essential part on the study of such major diseases. In mathematical point of view CKD has lot of parameter that involved like Hemoglobin, Creatinine, Blood Uric Acid, Blood Urea, GFR, and Volume of the Kidney, Sodium, Potassium and many more. Each of the parameter has essential work in human body for healthy nature. But one of the above mention is effected the human body are not behaving normally. So it is important to everyone for a healthy life to maintain each parameter's range keep remaining same as they have.

The terms that are commonly related with the CKD are fuzzy in natures. It is not clearly define that the only values of the above parameter which associates with CKD are sufficient to say a person is in trouble as it depending upon the nature of the body, diet of the person, and also the test KFT, So it is important to study the above parameter under the fuzzy environment to handle these critical situation. Which is developed by L. Zadeh (1965) to handle uncertainty, vagueness etc.

In this paper, we developed some new parameter $\alpha, \beta, \gamma, \delta$ which are co-related with the parameter Hemoglobin, Creatinine, Blood Uric Acid, Blood Urea, GFR. Existence of these are shown mathematically. A mathematical studied has been made under fuzzy environment. It is found that these parameters helps us to provide quite clear idea about either the person is suffering from CKD or not.

2. Definition of some Basic Mathematical terms :

Matrix : The knowledge of matrices is necessary in various branches of mathematics. Matrices are one of the most powerful tools in mathematics. This mathematical tool simplifies our work to a great extent when compared with other straight forward methods. The evolution of concept of matrices is the result of an attempt to obtain compact and simple methods of solving system of linear equations. Matrices are not only used as a

representation of the coefficients in system of linear equations, but utility of matrices far exceeds that use. Matrix notation and operations are used in electronic spreadsheet programs for personal computer, which in turn is used in different areas of business and science like budgeting, sales projection, cost estimation, analyzing the results of an experiment etc. Also, many physical operations such as magnification, rotation and reflection through a plane can be represented mathematically by matrices. Matrices are also used in cryptography. This mathematical tool is not only used in certain branches of sciences, but also in medical sciences, genetics, economics, sociology, modern psychology and industrial management.
Simply A matrix is an ordered rectangular array of numbers or functions. The numbers or functions are called the elements or the entries of the matrix.

Representation of a Matrix : if $a_{11}, a_{12}, a_{21}, a_{22}$ are the entries of a matrix then it can be represent by the following way

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, B = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \text{ and so on.}$$

In the above examples, the horizontal lines of elements are said to constitute, rows of the matrix and the vertical lines of elements are said to constitute, columns of the matrix. Thus A has 2 rows and 2 columns, B has 2rows and 3 columns.

Order of a matrix :A matrix having m rows and n columns is called a matrix of order m × n or simply m × n matrix (read as an m by n matrix). So referring to the above examples of matrices, we have A as 2 × 2 matrix, B as 2 × 3 matrix

Square Matrix :A matrix having same number of rows and columns i.e. n is called a square matrix of order n. So referring to the above examples of matrices, we have A is a square matrix.

Trace of Square Matrix: The sum of the diagonal elements of a square matrix is known as Trace of the matrix. Such as trace of the above matrix A, is define and denoted by

$$Tr(A) = a_{11} + a_{22}$$

Determinant of a Square Matrix: The determinant of a square matrix is a function whose domain is the square matrix to a real or complex number (depend on the real or complex entries) define below
$f: M \to K$, where K is the set of real number or set of complex number. As in this paper we are only dealing with a square matrix of order 2. So here with we only define the determinant of 2-order square matrix and it is denoted by $\det(A)$or $|A|$. Example, if
$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, then$\det(A) = a_{11}a_{22} - a_{12}a_{21}$

Basic of fuzzy set theory : Fuzziness is associated with subjective judgment. In the real world, complexity often arise from uncertainty in the form of ambiguity. The theory of probability has been age old and effective tool to handle uncertainty, but it can be applied only to situation whose characterized are based on random process. Uncertainty also arise due to partial information or information which are not reliable. Therefore fuzzy mathematics is an excellent tool to handle such vagueness. From the historical point of view the issue of uncertainty has not always been embraced within the scientific community. The leading theory which deal with uncertainty is probability theory but later on it changed by L.Zadeh (1965) to fuzzy set theory and then the application had started. Later on the theory had developed by Dubois and Prade (1980), kandel and lee (1979) and Kauffiman(1975).The characteristic function of a crisp set assigns a value either 1 or 0 to each object in a set and thereby distinguishes the members and non-members of crisp set under consideration.

The function can be generalized such that the values assigned to the elements of a set fall within a specific range from 0 to 1 and indicate the membership grade of these elements in the set in question. Larger values denote higher degree of set membership. Such function is called a membership function, and the set is known as fuzzy set. In short, it can be opinedthat the crisp set and logic divide the world of 'yes or no', 'true or false' but nothing in between. On the other hand, fuzzy sets and logic deal with objects that are of degree with all possible grades between 'yes or no'. Thus, fuzzy set represents the vague or ill-defined (not well-defined) concept like *good*, *very good*, *poor*, *intelligent*, *large*, *and medium large* etc., and hence, it can be extensively applied in a wide range of area. Zadeh (1965) developed this novel concept of fuzzy sets that created a new branch of Mathematics which is used to characterize the uncertainty. A lot of significant developments have been made by the researchers in the last five decades and applied it in a large variety of fields. It is now being applied in almost all branches of knowledge such as management science, information science, social sciences, process controlling, clustering, pattern recognition, decision making, biology, genetic coding, coding theory, medicine sciences, system theory, operational research, game theory, and many more. In the last three decades, significant process has been made in the development of the fuzzy set and fuzzy logic theory and their use in the large varieties of applied topics where uncertainty, vagueness and ambiguity are involved.

Definition 2.1:

Fuzzy set : Let X is a collection of objects denoted generally by x, then a fuzzy set of ordered pairs Ã in X is a set of order pairs

$$\tilde{A} = \{ (x, \mu_{\tilde{A}}(x)) : x \in X\}$$

$\mu_{\tilde{A}}$is called the membership function or grade of membership of x in Ã. The range of the membership function is a subset of the non-negative real number whose supremum is finite.

Definition 2.2:

Height of a fuzzy set: It is defined as the largest membership grade obtained by any element of a fuzzy set. i.e.

$$h(A) = sup_{x\in X}\mu_{A}(x).$$

Definition 2.3:

Normal fuzzy set: A fuzzy set A is said to be normal if $h(A) = 1$.

Definition 2.4:

Exponential fuzzy set: A fuzzy number $\hat{A} = [a, b, c, d]$ is called exponential if the membership function of the set is given by

$$\mu_{A}(x) = \begin{cases} e^{\frac{-(b-x)}{b-a}}, & a \le x \le b \\ 1, & b \le x \le c \\ e^{\frac{-(x-c)}{d-c}}, & c \le x \le d \end{cases}$$

Simply it can be express as $\hat{A} = [a, b, c, d]$

Zadeh extension principle: When a crisp function $f: X \to Y$is said to be fuzzified when it is extended to act on fuzzy set defined on X and Y. i.e

$$\tilde{f}: \tilde{X} \to \tilde{Y}$$

And its inverse has the form $\tilde{f}^{-1}: \tilde{Y} \to \tilde{X}$

The extension principle state that for a given crisp function $f: X \to Y$ induces two functions $\tilde{f}$ and $\tilde{f}^{-1}$which are defined above for which membership function are given by

$$[f(\tilde{A})](y) = sup_{x:y=f(x)}\mu_{\hat{A}}(x)$$

For all A ∈ $\tilde{X}$. And

$$[\tilde{f}^{-1}(B)](x) = \mu_{B}f(x)$$

For all B ∈ $\tilde{Y}$

3. Terms that related with CKD :

Haemoglobin: It is a type of globular protein present in red blood cells (RBC), which transports oxygen in our body through blood. It is a tetrameric protein and contain heme prosthetic group attached to each subunit. It is a respiratory pigment and helps in transporting oxygen as oxyhaemoglobin from the lungs to different parts of the body. Haemoglobin present in human body is coded by HBA1, HBA2 and HBB genes. The sequence of amino acids in polypeptide chains of Hb varies in different species. The heamoglobinlevel is measured in g/dL of the blood. For a healthy individual ranges from 13 g/dL to 18 g/dL and for woman from 11.5 g/dL to 16.5 g/dL .

There are various reasons for heamoglobin deficiency. Heamoglobin deficiency leads to the lower oxygen carrying capacity of the blood. It can be due to the nutritional deficiency, cancer, kidney failure or any genetic defect. The symptoms of low haemoglobin are shortness of breath, Dizziness, weakness, pale or yellow skin, chest pain etc.

GFR: Clinical assessment of kidney function is central to the practice medicine. GFR is widely accepted as the best index of kidney function in health and disease, and accurate values are require for optimal decision making. Estimated GFR based on serum creatinine is now widely reported by clinical laboratories. The range of GFR for a individual one is categorized as follows

Value of GFR	Stage
Greater than or equal90	Normal
60-89	Mild GFR
30-59	Moderate GFR
15-29	Severe GFR
Less than 15	Failer/ Dialysis

Blood urea: The urea cycle enzymes generate urea, these are mainly in the liver but are also ubiquitously expressed at low levels in other tissues. The metabolic process is altered in several conditions such as by diets, hormones, and diseases. Urea is then eliminated through fluids, especially urine. Blood urea nitrogen (BUN) has been utilized to evaluate renal function for decades. New roles for urea in the urinary system, circulation system, respiratory system, digestive system, nervous system, etc., were reported lately, which suggests clinical significance of urea.Blood urea for a healthy individual ranges from 9 mg/dL to 21 mg/dL for male and 7 mg/dL to 19 mg/dL.

Blood uric acid: Uric acid is a normal body waste product. It forms when chemicals called purines break down. Purines are a natural substance found in the body. They are also found in many foods such as liver, shellfish, and alcohol. They can also be formed in the body when DNA is broken down. When purines are broken down to uric acid in the blood, the body gets rid of it when you urinate or have a bowel movement. But if your body makes too much uric acid, or if your kidneys aren't working well, uric acid can build up in the blood. Uric acid levels can also increase when you eat too many high-purine foods or take medicines like diuretics, aspirin, and niacin. Then crystals of uric acid can form and collect in the joints. This causes painful inflammation. This condition is called gout. It can also lead to kidney stones. For a healthy individual ranges from 3.5 mg/dL to 7 mg/dL and for woman from 2.6 mg/dL to 6.0 mg/dL .

Serum Creatinine : The serum creatinine concentration is widely interpreted as a measure of the glomerular filtration rate (GFR) and is used as an index of renal function in clinical practice. Glomerular filtration of creatinine, however, is only one of the variables that determines its concentration in serum. Alterations in renal handling and metabolism of creatinine and methodological interferences in its measurement may have a profound impact on the serum concentration of creatinine. The normal range of healthy man and woman are [0.6 − 1.2] and [0.6 − 1.1]for the Serum creatinine respectively.

4. Introducing some Basic parameters:

(a) $\boldsymbol{\alpha = \frac{Cr}{GFR}}$ **,** it is the ratio between Serum creatinine and theglomerular filtration rate (GFR).

(b) $\boldsymbol{\beta = \frac{Cr}{Hb}}$**,** It is the ratio between Serum creatinine and theHaemoglobin.

(c) $\boldsymbol{\gamma = \frac{Cr}{Blood\ Urea}}$ **,** It is the ratio between Serum creatinine and theBlood urea.

(d) $\boldsymbol{\delta = \frac{Cr}{Blood\ Uric\ acid}}$**,** It is the ratio between Serum creatinine and theBlood Uric acid.

5. Properties of the parameters , $\boldsymbol{\beta, \gamma, \delta}$ **:**

1. For a living human being as $Cr \neq 0, GFR \neq 0, Hb \neq 0, Blood\ Urea \neq 0$and $ood\ Uric\ Acid \neq 0$, so all the parameter introduce above are well defined.
2. For a healthy man
   (a) **The range of** $\boldsymbol{\alpha}$ **is**$[\mathbf{0.00666, 0.0133}]$,
   Proof: the range of Serum creatinine $Cr = [0.7,1.2\,]$, and GFR ≥ 90
   Now, $\alpha = \frac{Cr}{GFR} = \frac{[0.7,1.2\,],}{90} = [\,0.0077, 0.0133]$
   As, GFR is always greater than 90, so by definition of α , It is clear that $\alpha \geq 0.0133$,
   (b) **The rangeof** $\boldsymbol{\beta}$ **is** $[\mathbf{0.0388, 0.0923}]$**,**
   Proof: The range of Serum creatinine $Cr = [0.7,1.2\,]$, and **Haemoglobin**= $[\,\mathbf{13, 18}]$
   Now, $\beta = \frac{Cr}{Hb} = \frac{[0.7,1.2\,],}{[\,13,18]} = [\,min\ L, max\ L] = [0.0388, 0.0923]$
   Where $L = \left\{\frac{0.7}{13}, \frac{1.2}{13}, \frac{0.7}{18}, \frac{1.2}{18}\right\} = \{0.04615, 0.923, 0.0333, 0.6666\}$
   As, Hb is always less than 18, so by definition of β , It is clear that $\beta \geq 0.0388$,

   (c) **The range of** $\boldsymbol{\gamma}$ **is**$[\mathbf{0.0333, 0.1333}]$
   Proof :The range of Serum creatinine $Cr = [0.7,1.2\,]$, and Blood urea= [9,21]
   Now, $\gamma = \frac{Cr}{Bl.\ Urea} = \frac{[0.7,1.2\,],}{[\,9,21]} = [\,min\ M, max\ M] = [0.0333, 0.1333]$
   Where $M = \left\{\frac{0.7}{9}, \frac{1.2}{9}, \frac{0.7}{21}, \frac{1.2}{21}\right\} = \{0.07778, 0.0333, 0.1333, 0.05714\}$

   (d) The range of $\boldsymbol{\delta}$ **is [0.08571, 0.17142]**
   Proof: The range of Serum creatinine $Cr = [0.7,1.2\,]$, and
   Blood uric Acid= [3.5,7]
   Now, $\delta = \frac{Cr}{Bl.\ Uric\ Acid} = \frac{[0.7,1.2\,],}{[\,3.5,7]} = [\,min\ N, max\ N] = [0.1, 0.2]$
   Where $N = \left\{\frac{0.7}{3.5}, \frac{1.2}{3.5}, \frac{0.7}{7}, \frac{1.2}{7}\right\} = \{0.2, 0.03428, 0.1714, 0.1\}$

3. For a healthy woman
   (a) The range of $\boldsymbol{\alpha}$ **is** $[\,\mathbf{0.00667, 0.01222}]$**,**
   Proof: the range of Serum creatinine $Cr = [0.6,1.1\,]$, and GFR ≥ 90
   Now, $\alpha = \frac{Cr}{GFR} = \frac{[0.6,1.1\,],}{90} = [\,0.00667, 0.0122]$
   As, GFR is always greater than 90, so by definition of α , It is clear that $\alpha \geq 0.0122$
   (b) The range of $\boldsymbol{\beta}$ **is** $[\mathbf{0.03636, 0.0956}]$
   Proof: The range of Serum creatinine $Cr = [0.6,1.2\,]$, and
   Haemoglobin= [11.5,16.5]
   Now, $\beta = \frac{Cr}{Hb} = \frac{[0.6,1.1\,],}{[\,11.5,16.5]} = [\,min\ L, max\ L] = [0.03636, 0.0956]$
   Where $L = \left\{\frac{0.6}{11.5}, \frac{1.1}{11.5}, \frac{0.6}{16.5}, \frac{1.1}{16.5}\right\} = \{0.0521, 0.03636, 0.0956, 0.6667\}$

As, Hb is always less than 16.5, so by definition of β , It is clear that $\beta \geq 0.03636$,

(c) The range of γ is [$0.0315, 0.1517$]

Proof : The range of Serum creatinine $Cr = [0.6,1.1]$,
and Blood urea= [7,19]

Now, $\gamma = \frac{Cr}{Bl.\ Urea} = \frac{[0.6,1.1\ ],}{[\ 7,19]} = [\,min\ M, max\ M] = [0.0315, 0.1517]$

Where $M = \left\{\frac{0.6}{7}, \frac{1.1}{7}, \frac{0.6}{19}, \frac{1.1}{19}\right\} = \{0.08571, 0.0315, 0.1571, 0.0578\}$

(d) The range of δ is [$0.1, 0.42307$]

Proof : The range of Serum creatinine $Cr = [0.6,1.1\ ]$, and
Blood uric Acid= [2.6,6.0]

Now, $\delta = \frac{Cr}{Bl.\ Uric\ Acid} = \frac{[0.6,1.1\ ],}{[\ 2.6,6]} = [\ min\ N, max\ N] = [0.1, 0.42307]$

Where $N = \left\{\frac{0.6}{2.6}, \frac{1.1}{2.6}, \frac{0.6}{6}, \frac{1.1}{6}\right\} = \{0.2307, 0.1, 0.42307, 0.1833\}$

4. Construction of Creatinine Matrix :

Using the parameter $\alpha, \beta, \gamma, \delta$ construct a matrix of order 2 which denoted as $[Cr]$ and define as

$$[Cr^{M/F}]_{min/max} = \begin{bmatrix} \alpha & \gamma \\ \delta & \beta \end{bmatrix}$$

Here, M /F represent for Male or Female and
Min or Max represent min or maximum values of the corresponding parameter.
Without loss of generality we are dealing with $[Cr^M]$ only.
Now,

$$[Cr^M]_{min} = \begin{bmatrix} \alpha & \gamma \\ \delta & \beta \end{bmatrix} = \begin{bmatrix} 0.0077 & 0.033 \\ 0.1 & 0.0388 \end{bmatrix}$$

$$[Cr^M]_{max} = \begin{bmatrix} \alpha & \gamma \\ \delta & \beta \end{bmatrix} = \begin{bmatrix} 0.0133 & 0.1333 \\ 0.2 & 0.0923 \end{bmatrix}$$

Now both the matrix $[Cr^M]_{min}$ and $[Cr^M]_{max}$ are non-singular, as determinant of both the matrix are non-zero. i.e

Det. $([Cr^M]_{min}) = \alpha\beta - \gamma\delta = (0.0077 * 0.0388) - (0.1 * 0.033) = -0.0030312 \neq 0$

Similarly

Det. $([Cr^M]_{max}) = \alpha\beta - \gamma\delta = (0.0133 * 0.0923) - (0.2 * 0.1333) = -0.025435 \neq 0$

Again if we find Trace of these matrix, then

Trace. $([Cr^M]_{min}) = \alpha + \beta = 0.0077 + 0.0388 = 0.0465$

Trace. $([Cr^M]_{max}) = \alpha + \beta = (0.0133 + 0.0923) = 0.1056$

5. Result and Discussion

The range of the Det. $[Cr^M] = [\,-0.025435, -0.0030312]$ and Trace $[Cr^M] = [0.0465, 0.1056,]$, the physical interpretation of these values indicates that for a healthy man we always find of Det. $[Cr^M]$ and Trace $[Cr^M]$ in between this.

As these values are depends on various parameter like lab testing, food consumption, ecology of the person etc. due to these uncertainty arises. According to Diniz et al. (2001), the uncertainty can also arise in the experiment part, data collection, measurement process as well as when determining the initial values. It is encountered that there are several fundamentally different types of uncertainty and there is enough scope of study for proper interpretation of the physical phenomena. A mathematical formulation in which these uncertainties can be properly characterized and investigated is now available in terms of Fuzzy sets theory (FST) proposed by Zadeh (1965).Applying Zadeh

Extension rule in Det. $[Cr^M]$ and Trace $[Cr^M]$, we have Det. $\widehat{[Cr^M]}$ and Tr $\widehat{[Cr^M]}$, and the fuzziied valued are given below

Det. $\widehat{[Cr^M]} = [-0.025435, -0.0030312] = \hat{A}$(say) and

Tr $\widehat{[Cr^M]} = [0.0465, 0.1056] = \hat{B}$(say)

Without loss of generality consider $\hat{A}$ *and* $\hat{B}$ are exponential fuzzy number and dividing four equal parts we have

$\hat{A} =$ ` $[-0.025435, -0.01983405, -0.00863215, -0.0030312]$

$\widehat{B} = [0.0465, 0.0662, 0.0859, 0.1056]$

For the fuzzy number $\hat{A} = [-0.025435, -0.01983405, -0.00863215 - 0.0030312]$is the membership function of the set is given by

$$\mu_A(x) = \begin{cases} e^{\frac{-(-0.01983405-x)}{-0.01983405-(-0.025435)}}, & -0.025435 \le x \le -0.01983405 \\ 1, & -0.01983405 \le x \le -0.00863215 \\ e^{\frac{-(x-(-0.00863215))}{-0.0030312-(-0.00863215)}}, & -0.00863215 \le x \le -0.0030312 \end{cases}$$

Similarly, For the fuzzy number $\widehat{B} = [0.0465, 0.0662, 0.0859, 0.1056]$is the membership function of the set is given by

$$\mu_B(x) = \begin{cases} e^{\frac{-(0.0662-x)}{0.0662-0.0465}}, & 0.0465 \le x \le 0.0662 \\ 1, & 0.0662 \le x \le 0.0859 \\ e^{\frac{-(x-0.0859)}{0.1056-0.0859}}, & 0.0859 \le x \le 0.1056 \end{cases}$$

Let us consider an example for a Patient if we get Det. $[Cr^M] = [-0.01535, -0.00902]$ and Trace $[Cr^M] = [0.065, 0.096,]$, then using membership function of above expression we have (Taken the minimum value)

$\mu_A(x) = 0.5$and$\mu_B(x) = 0.4$, these values indicate the patient approaches to the CKD, higher membership values indicates healthy person.

6. Conclusion

The application of fuzzy set theory is now a broad subject of analysis. In this paper we are attempting to a Mathematical approaches for treating a CKD patient under fuzzy environment. Some new parameter are define and justifying through our new methods. In near future this paper helps a new mathematical ways of finding kidney damage or improve of kidney.

7. References

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