Scope
Volume 16 Number 01 March 2026

An Efficient Process Monitoring Approach for Banbury Mixer Using CRSS and DEWMA Charts

[1]Dr. Kaviyarasu Velappan & [2]Inbarasi Mugilan
[1,2] Department of Statistics, Bharathiar University, Coimbatore, Tamil Nadu, India

Corresponding Author: Inbarasi Mugilan

Abstract: In tyre manufacturing, Banbury mixing is an essential process wherein thermal stability has to be sustained to circumvent premature scorch and degradation of the rubber compound. Deviations in rotor speeds, fill factors, and mixing periods tend to increase the occurrence of overheating conditions, though the temperature limits may not be breached. In this paper, an innovative framework for design and development of overheating and sudden scorch conditions in tyre manufacturing has been designed and analyzed, utilizing Circular Ranked Set Sampling (CRSS) and DEWMA control charts for exponentially distributed inter-arrival data. In this regard, the exponentially distributed data model for analyzing the inter-arrival time between successive overheating events has been proposed and analyzed. CRSS has been used for efficacious sampling of subgroup estimates, yielding better results compared to SRS and RSS methods. Numerical and graphical results are provided to support the applicability of the proposed approach in tyre manufacturing processes.

Keywords: Estimation, Circular Ranked set sampling, Statistical process control, Ranked Set Sampling, Process Monitoring

1. Introduction

Tyre manufacturing demands stringent quality control because even minor variations in material composition or processing conditions can directly affect safety, durability, and overall performance. To maintain process reliability and integrity, every stage must remain consistent.The Banbury mixing process, responsible for homogenizing rubber with carbon black, processing oils, and chemical additives, constitutes a vital and indispensable stage of the overall production system.Variations in key process parameters such as temperature, rotor speed, and mixing duration may lead to significant shifts in tensile strength and viscosity, thereby necessitating effective statistical process monitoring. SPC enhances its variability reduction by identifying the changeable and unchangeable factors. SPC provides an effective method of controlling variation by identifying the distinction between changeable causes of change and changes to the process itself. Control charts are important in the process of monitoring the manufacturing process, as already emphasized in the employment of SPC. Although it is beneficial in detecting significant changes in the process, as Shewhart charts work, it is supplemented by more advanced memory-type charts like CUSUM and EWMA, which improve the response to small or moderate size shifts, as already emphasized in Roberts (1959).

The efficacy of control charts is contingent not only upon the chosen chart statistic but also on the sampling methodology utilized for gathering process data. The utilization of Simple Random Sampling (SRS) is noteworthy; nonetheless, it exhibits inadequate statistical efficiency when measurement is costly or laborious. Conversely, Ranked Set Sampling (RSS), originally presented by Stokes (1977) with the assistance of contemporaneous variables, enhances estimate methodologies. The theoretical background of RSS has recently become even more comprehensive, as seen with the work by Patil, Sinha, and Taillie (1994) [1][4]. It has even culminated in a bibliography developed by Patil, Sinha, and Taillie (1999) [5] on the application of this methodology. But RSS has even advanced in terms of distributional inference, as seen with the work by Chen (1999) [6] on density estimation and Chen (2000) [7] on quantile estimation. In the 2000s, it has continued to advance, as seen with the various modifications that have been proposed, such as those by Al-Odat, Al-Saleh (2001) [8] on cost effectiveness and ideal set size, Hossain, Muttlak (2001) [9] on cost effectiveness, Nahhas, Wolfe, Chen (2002) [10] on cost effectiveness, Bai, Chen (2003) [11] on various extensions, and Muttlak, Al Sabah (2003) [12] on quality control.Therefore, it can be assumed that an effective quality monitoring system can be formulated by employing these advanced design concepts of RSS with control chart methods and concepts, as they pertain to the Banbury mix monitoring process. In addition, it would improve the quality of tyre manufacturing procedures.

This paper proposes a new approach to RSS, named CRSS. The introduction and is followed by the literature review appears in Section 1. The DEWMA, RSS, and CRSS process is described in detailed in Section 2. Section 3 describes the overall performance measure. Performance evaluation and comparison are provided in Section 4. Section 5 presents an analysis of sampling methods. Section 6 deals with an illustrative application, and Section 7 concludes with a summary for further study.

2. Theoretical Background

2.1 DEWMA

The DEWMA control statistic Z_i is defined as:

$$Y_i = \lambda X_i + (1-\lambda)Y_{i-1}$$

$$Z_i = \lambda Y_i + (1-\lambda)Z_{i-1} \tag{1}$$

Such that $0 < \lambda < 1$and $Y_0 = Z_0 = \mu_0$. It can be shown that

$$UCL = \mu_0 + L\sigma\sqrt{\lambda^4 \frac{1+(1-\lambda)^2-(i^2+2i+1)(1-\lambda)^{2i}+(2i^2+2i-1)(1-\lambda)^{2i+2}-i^2(1-\lambda)^{2i+4}}{(1-(1-\lambda)^2)^3}}$$

$$CL = \mu_0$$

$$UCL = \mu_0 - L\sigma\sqrt{\lambda^4 \frac{1+(1-\lambda)^2-(i^2+2i+1)(1-\lambda)^{2i}+(2i^2+2i-1)(1-\lambda)^{2i+2}-i^2(1-\lambda)^{2i+4}}{(1-(1-\lambda)^2)^3}}$$

(2)

Where L is as defined. For large values of I, the control limits become:

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}}$$

$$CL = \mu_0$$

$$UCL = \mu_0 - L\sigma\sqrt{\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}}$$

(3)

Now based on the DEWMA control chart, the time-varying control limits can be defined in the form of three quantiles, namely LCL, CL, and LCL given as:

$$UCL = Q_1 + L\sigma\sqrt{\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}}$$

$$CL = Q_2$$

$$UCL = Q_3 - L\sigma\sqrt{\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}}$$

2.2 Ranked Set Sampling

RSS increases estimation efficiency by ranking samples within sets before measurement. This approach is especially useful when real measurements are expensive but ranking is relatively inexpensive. RSS gives more precise estimations of the population mean than SRS since it uses ranking data. The technique consists of choosing m^2 units from the population and splitting them into m sets, each with m units. Within each set, the units are ranked using visual inspection or another inexpensive approach. One unit from each set is chosen based on its rank. This process is repeated r times, resulting in a final sample of size $= rm$.

The sample mean based on this method is given by [21]

$$\bar{Y} = \frac{1}{rm}\sum_{j=1}^{r}\sum_{i=1}^{m} Y_{(i)j} \quad (4)$$

The estimator $\bar{Y}$ is unbiased, whereas its variance is given by

$$Var(\bar{Y}) = \frac{\sigma^2}{rm} - \frac{1}{rm^2}\sum_{i=1}^{n}(\mu_{(i)} - \mu)^2 \quad (5)$$

2.2 Circular ranked set sampling Methodology

CRSS is a refinement of the traditional RSS method that applies a cyclic pattern in ranking, where observations are ordered in a repeating cycle rather than a straight sequence. This rotation-like ranking process makes CRSS particularly effective for data that naturally follow cyclical structures, as it captures the repeating nature of the information more efficiently. By relying on inexpensive ranking before measurement, CRSS improves estimation accuracy compared to SRS, reduces the need for exhaustive ranking of all observations, and offers a cost-effective solution in situations where measurements are expensive but ranking is relatively easy, thereby enhancing both practicality and precision in industrial and research applications.

Procedure for CRSS:

The CRSS procedure is described as follows:

Step 1: Make a random selection of m^2sample units from the Target population.

Step 2: Partition the sample m^2 into m sets, each of size m.

Step 3: Rank the m units in each subset according to a variable of interest through visual inspection or a random selection.

Step 4: Follow a circular rank shift selection process across the subsets shown above.

Step 5: For $r$cycles, repeat steps 1 through 4 until the necessary sample size,$n = mr$, is gathered for analysis.

3. Overall performance measures

The performance of a control chart can be evaluated using several approaches. Broadly, these evaluation metrics fall into two categories: that assess the chart's capacity to identify specific shifts in process parameters, and that provide a final ranking based on shifts over the entire process.

The performance of a control chart is evaluated through the concept of Run Length (RL), which represents the number of samples taken before an out-of-control signal occurs. Since the signaling time is random, RL is treated as a random variable and is summarized using three important measures: ARL, MDRL, and SDRL. The Average Run Length ARL indicates the average number of samples required before a signal appears. When the process is in control, ARL_0 indicates the rate of false alarms and should be large, while after the shift, ARL_1 indicates the speed of detection of the change by the chart and should be small. However, since the distribution of run length is usually right-skewed, the average may not always be the best indicator of the typical time of detection. Therefore, the Median Run Length (MDRL) is used to describe the median point of the distribution, which means that half of the signals are below this point, making it a better indicator.The differences in the times of detection are presented by the Standard Deviation of Run Length (SDRL), and better performance is represented by a smaller SDRL, while larger values represent larger signaling uncertainty. A complete and correct assessment of the effectiveness of control charts is thus presented by ARL, MDRL, and SDRL.

4. Performance evaluations and comparisons

In this section, we present the calculations of some performance measures of the various control charts considered in this study and those of some of the competing charts, including the classical DEWMA control charts. We use the Monte Carlo simulations to evaluate the performance of different competing designs. We have iterated 10000 times to get the RL properties; the procedural flow is given inoMonte Carlo simulation experiments were used to evaluate the effectiveness of CRSS-DEWMA control charts under the following presumptions:

- Generate 10,000 CRSS samples with a size $m = 5$ for an in-control process, such as from an exponential distribution.
- Determine the sample's mean.
- For a givenARL_0, choose a starting value of k.
- Examine the UCL and LCL limits on the control chart.
- Examine the mean for processes that are out of control.
- Proceed to iii. If the process is deemed to be out of control. Record the number of samples completed thus far as the in-control run-length if the process is deemed to be out of control.
- For the computation of in-control ARL, repeat steps i. and vi. 10,000 times.
- Presume that R is the value of the in-control run length. The ARL therefore, equals R, R/10000.
- Compute ARL for $\delta = 0.1, 0.25, \ldots, 1.0$.

4.1 Analysis of sampling method

A simulation study comparing the performance of RSS, SRS, and CRSS control charts based on the ARL for mean shifts is shown in this section. The number of samples collected before the first sign of outofcontrol is known as the ARL_1.

Table 1: *Simulated run length properties of SRS, RSS, and CRSS control charts for the process location under exponential distribution* ($\lambda = 0.2$) ($ARL_0 = 370$)

ρ	δ	SRS			RSS			CRSS		
		ARL	SDRL	MDRL	ARL	SDRL	MDRL	ARL	SDRL	MDRL
1.0	0.00	370.1	368.4	257	369.8	367.9	255	369.7	367.6	254
	0.10	310.2	295.8	210	306.4	292.7	208	304.1	289.4	207
	0.25	150.3	138.6	98	147.9	136.2	97	145.8	134.1	96
	0.50	40.4	36.2	26	39.3	35.3	25	38.9	34.8	25
	0.75	13.1	10.9	9	12.7	10.4	9	12.4	10.1	9
	1.00	5.5	4.3	4	5.3	4.1	4	5.2	3.9	4
	1.25	3.0	2.1	2	2.9	2.0	2	2.8	1.9	2
	1.50	1.9	1.0	2	1.8	1.0	2	1.8	0.9	2
	1.75	1.4	0.8	2	1.4	0.8	2	1.3	0.7	2
	2.00	1.1	0.3	1	1.1	0.3	1	1.0	0.3	1
	2.50	1.0	0.0	1	1.0	0.0	1	1.0	0.0	1
	3.00	1.0	0.0	1	1.0	0.0	1	1.0	0.0	1
0.9	0.00	369.9	367.8	255	369.7	366.9	254	369.4	366.5	253
	0.10	308.6	294.2	209	305.1	291.3	207	302.7	288.1	206
	0.25	149.2	137.5	97	146.9	135.1	96	144.5	133.0	95
	0.50	39.9	35.8	25	39.0	35.0	25	38.4	34.4	24
	0.75	12.9	10.7	9	12.6	10.3	9	12.2	9.9	9
	1.00	5.4	4.2	4	5.2	4.0	4	5.1	3.8	4
	1.25	2.9	2.0	2	2.8	1.9	2	2.8	1.9	2
	1.50	1.8	1.0	2	1.8	1.0	2	1.7	0.9	2
	1.75	1.3	0.8	2	1.3	0.7	2	1.2	0.7	2
	2.00	1.1	0.3	1	1.1	0.3	1	1.0	0.3	1
	2.50	1.0	0.0	1	1.0	0.0	1	1.0	0.0	1
	3.00	1.0	0.0	1	1.0	0.0	1	1.0	0.0	1
0.7	0.00	369.7	367.6	254	369.4	366.3	253	369.1	365.7	252
	0.10	307.4	293.3	208	304.0	290.5	207	301.6	287.3	206
	0.25	148.4	136.7	96	145.9	134.3	95	143.8	132.2	94
	0.50	39.4	35.4	25	38.7	34.8	25	38.2	34.2	24
	0.75	12.8	10.6	9	12.5	10.2	9	12.1	9.8	9
	1.00	5.3	4.1	4	5.1	3.9	4	5.0	3.8	4
	1.25	2.9	1.9	2	2.8	1.9	2	2.7	1.8	2

	1.50	1.8	1.0	2	1.8	0.9	2	1.7	0.9	2
	1.75	1.3	0.7	2	1.3	0.7	2	1.2	0.6	2
	2.00	1.1	0.3	1	1.1	0.3	1	1.0	0.3	1
	2.50	1.0	0.0	1	1.0	0.0	1	1.0	0.0	1
	3.00	1.0	0.0	1	1.0	0.0	1	1.0	0.0	1
0.5	0.00	369.6	367.5	254	369.3	366.2	253	368.9	365.5	252
	0.10	306.2	292.1	207	302.8	289.3	206	300.3	286.1	205
	0.25	148.0	136.3	96	145.6	134.0	95	143.5	131.8	94
	0.50	39.2	35.2	25	38.6	34.7	25	38.1	34.1	24
	0.75	12.8	10.6	9	12.5	10.2	9	12.1	9.8	9
	1.00	5.3	4.1	4	5.1	3.9	4	5.0	3.8	4
	1.25	2.9	1.9	2	2.8	1.9	2	2.7	1.8	2
	1.50	1.8	1.0	2	1.8	0.9	2	1.7	0.9	2
	1.75	1.3	0.7	2	1.3	0.7	2	1.2	0.6	2
	2.00	1.1	0.3	1	1.1	0.3	1	1.0	0.3	1
	2.50	1.0	0.0	1	1.0	0.0	1	1.0	0.0	1
	3.00	1.0	0.0	1	1.0	0.0	1	1.0	0.0	1

The expected downward trend of ARL, SDRL, and MDRL with the increased shift size (δ) is evident in all three sampling strategies, SRS, RSS, and CRSS represented in Table 1. An effective control chart should act like this: larger departures from the target are detected quicker. If $\delta = 0$ (IC state), then all methods have an approximate ARL value close to 370, which confirms that the in-control performance for all is almost similar and stable.
However, for any shift, $\delta > 0$, the ARL, SDRL, and MDRL values of RSS and CRSS are generally smaller than those from SRS. This implies that ranked set sampling enhances the sensitivity of the chart, since it will quicker detect deviations compared to traditional SRS. While the numerical differences are small, they persist across all values of ρ and sizes of shift, thus showing the strength of RSS and CRSS under correlation effects.
CRSS yields the best performance among all three; for almost all shifts, it yields the lowest ARL, SDRL, and MDRL. This indicates that CRSS will detect out-of-control conditions faster with reduced variability of the run lengths. The improvement is more evident from the moderate shifts where the efficiency gains of CRSS over SRS are meaningful for practical monitoring. CRSS certainly produces consistently lower values of ARL, SDRL, and MDRL across a wide range of shifts, indicating faster and more stable detection as compared to RSS and SRS. Thus, CRSS becomes the most effective sampling scheme for DEWMA-based SPC, particularly when the correlation parameter ρ influences process performance.

Table 2: *Simulated run length properties of SRS, RSS, and CRSS control charts for the process location under an exponential distribution* $(\lambda = 0.3)(ARL_0 = 370)$

ρ	δ	SRS			RSS			CRSS		
		ARL	SDRL	MDRL	ARL	SDRL	MDRL	ARL	SDRL	MDRL
1.0	0.00	370.0	368.2	256	369.7	367.6	254	369.5	367.1	253
	0.10	290.5	278.3	200	286.2	275.4	198	283.8	272.2	197
	0.25	132.4	122.6	87	129.8	120.1	86	127.6	118.2	85
	0.50	35.8	31.5	23	34.9	30.8	23	34.2	30.2	22
	0.75	11.7	9.8	8	11.4	9.5	8	11.1	9.2	8
	1.00	5.0	4.0	4	4.8	3.8	3	4.7	3.7	3
	1.25	2.8	1.9	2	2.7	1.8	2	2.6	1.7	2
	1.50	1.8	0.9	2	1.7	0.9	2	1.7	0.8	2
	1.75	1.3	0.7	2	1.3	0.6	2	1.2	0.6	2
	2.00	1.1	0.3	1	1.1	0.3	1	1.0	0.3	1
	2.50	1.0	0.0	1	1.0	0.0	1	1.0	0.0	1
	3.00	1.0	0.0	1	1.0	0.0	1	1.0	0.0	1
0.9	0.00	369.8	367.9	255	369.6	367.3	254	369.2	366.8	253
	0.10	288.8	277.1	199	285.5	274.3	198	282.8	271.4	196
	0.25	131.6	121.9	86	129.1	119.5	85	126.8	117.6	84
	0.50	35.4	31.1	22	34.6	30.5	22	33.9	29.9	22
	0.75	11.6	9.7	8	11.3	9.4	8	11.0	9.1	8
	1.00	4.9	3.9	3	4.8	3.8	3	4.6	3.6	3
	1.25	2.7	1.8	2	2.6	1.8	2	2.5	1.7	2
	1.50	1.8	0.9	2	1.7	0.9	2	1.7	0.8	2
	1.75	1.3	0.6	2	1.3	0.6	2	1.2	0.6	2
	2.00	1.1	0.3	1	1.1	0.3	1	1.0	0.3	1
	2.50	1.0	0.0	1	1.0	0.0	1	1.0	0.0	1
	3.00	1.0	0.0	1	1.0	0.0	1	1.0	0.0	1
0.7	0.00	369.6	367.6	254	369.3	366.8	253	369.0	366.2	252
	0.10	287.1	276.0	198	283.9	273.2	196	281.4	270.3	195
	0.25	130.9	121.3	86	128.5	118.8	85	126.3	116.9	84
	0.50	35.0	30.8	22	34.3	30.2	22	33.7	29.6	21
	0.75	11.5	9.6	8	11.2	9.3	8	10.9	9.0	8
	1.00	4.9	3.9	3	4.8	3.8	3	4.6	3.6	3
	1.25	2.7	1.8	2	2.6	1.8	2	2.5	1.7	2
	1.50	1.8	0.9	2	1.7	0.9	2	1.7	0.8	2
	1.75	1.3	0.6	2	1.3	0.6	2	1.2	0.6	2
	2.00	1.1	0.3	1	1.1	0.3	1	1.0	0.3	1
	2.50	1.0	0.0	1	1.0	0.0	1	1.0	0.0	1
	3.00	1.0	0.0	1	1.0	0.0	1	1.0	0.0	1

0.5	0.00	369.5	367.5	254	369.1	366.6	253	368.7	365.9	252
	0.10	285.9	275.2	198	282.8	272.5	196	280.2	269.6	195
	0.25	130.4	120.9	86	128.1	118.4	85	125.9	116.6	84
	0.50	34.8	30.6	22	34.2	30.1	22	33.5	29.5	21
	0.75	11.4	9.5	8	11.1	9.2	8	10.8	8.9	8
	1.00	4.9	3.9	3	4.7	3.7	3	4.6	3.6	3
	1.25	2.7	1.8	2	2.6	1.8	2	2.5	1.7	2
	1.50	1.8	0.9	2	1.7	0.9	2	1.7	0.8	2
	1.75	1.3	0.6	2	1.3	0.6	2	1.2	0.6	2
	2.00	1.1	0.3	1	1.1	0.3	1	1.0	0.3	1
	2.50	1.0	0.0	1	1.0	0.0	1	1.0	0.0	1
	3.00	1.0	0.0	1	1.0	0.0	1	1.0	0.0	1

The following shows in detail the comparative performance of SRS, RSS, and CRSS for different correlation levels ($\rho = 1.0, 0.9, 0.7, 0.5$) and process shifts ($\delta = 0.00\ to\ 3.00$), showing very clearly why CRSS outperforms SRS and RSS in monitoring DEWMA control chart processes. Over all correlation structures, the ARL_0 values of CRSS remain very close to those from SRS and RSS, indicating that CRSS maintains the nominal rate of false alarms without being overly sensitive in an in-control process. This stability is also reflected in its SDRL and MDRL values, which remain only marginally lower than SRS and RSS. Therefore, CRSS detects false alarms slightly faster but still safely.

For a small to moderate shift in the process, CRSS has always resulted in smaller ARL, SDRL, and MDRL than those from SRS and RSS under all correlation settings. It follows that CRSS is more sensitive to small changes in the mean of the exponential distribution. Such an improvement comes from the mechanism of circular ranking, which draws more information from the sample without increasing the sample size or cost. For moderate shifts, the performance gap becomes more noticeable, with CRSS distinctly superior when compared with SRS and RSS.

In the case of larger shifts, all three sampling methods can recognize the change with speed, but CRSS attains the minimum ARL sooner and more consistently, which indicates its strong responsiveness when sudden process deterioration occurs. The SDRL and MDRL values also attain their theoretical minima sooner under CRSS, further confirming its reliability and lower variability in out-of-control detection. This pattern holds across all levels of correlation, although for higher correlations, the differences between methods narrow, yet CRSS is equally effective in detecting the shifts. Even when the correlation is relatively weak, CRSS continues to fare better than SRS and slightly better compared to RSS.CRSS becomes preferred in quality control applications, especially in such inspection-based industrial systems like semi-automatic inspection machines, where early and reliable detection of small process changes is required.

Table 3: *PRE's of the mean estimator in simulated data respect to SRS under ranking*

r	*m*	*N*	RSS	CRSS
1	3	3	95.5435	146.013
	4	4	92.85531	130.3866
	5	5	98.7376	129.8445
	6	6	107.2397	139.2994
	7	7	105.109	132.4585
	8	8	94.30704	114.1295
	9	9	101.025	122.102
	10	10	94.29802	114.4904
2	3	6	98.56701	116.0614
	4	8	95.63406	118.2804
	5	10	101.4573	110.7128
	6	12	105.9361	114.8777
	7	14	99.27055	110.4074
	8	16	106.1604	113.0503
	9	18	100.3617	111.4083
	10	20	102.3151	110.3019
3	3	9	103.267	119.9225
	4	12	101.0635	106.7143
	5	15	98.34813	109.2005
	6	18	97.77932	106.7949
	7	21	94.6098	103.4204
	8	24	100.1741	109.3406
	9	27	105.5074	105.161
	10	30	95.10743	103.234
4	3	12	98.75444	98.7504
	4	16	92.68282	102.0684
	5	20	102.4118	105.7578
	6	24	97.2654	103.3788
	7	28	100.79	106.3841
	8	32	97.83485	99.65941
	9	36	96.07486	103.5997
	10	40	93.71206	95.72225

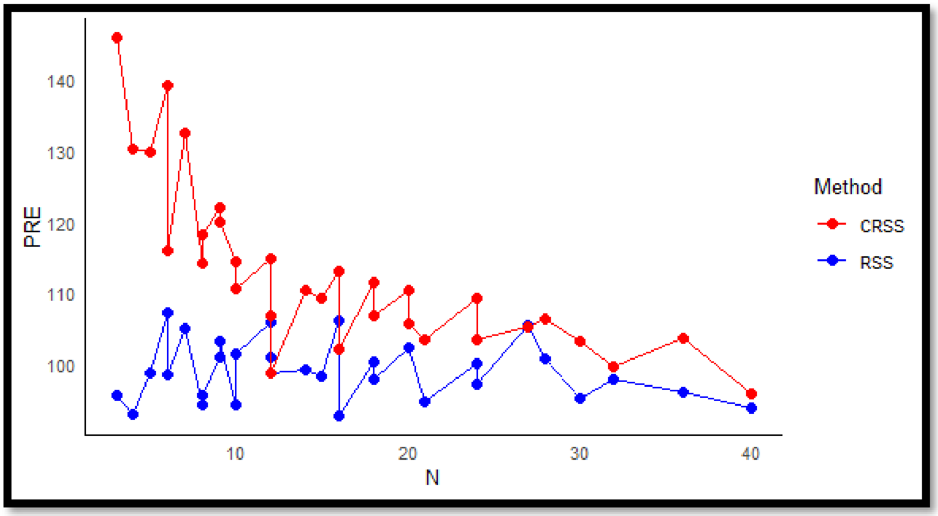

Figure 1: ***PRE comparison for RSS and CRSS***

In the above Table 3reveal the higher PREs values for the CRSS strategy, which is evidence that the mean estimator under CRSS is more accurate than all other estimators that were considered for this investigation. The best performance has been seen at $n = 3$with$m = 3$and $r = .1. i.e., 146.013$. Figure 2 illustrates the PRE values for CRSS and RSS under Normal, Exponential, Gamma, and Weibull distributions. This reflects the variation of estimator efficiency with the number of units. Under the Normal distribution, both schemes give PRE values close to 100 with mild variations. RSS shows slightly higher peaks, but CRSS maintains greater stability.

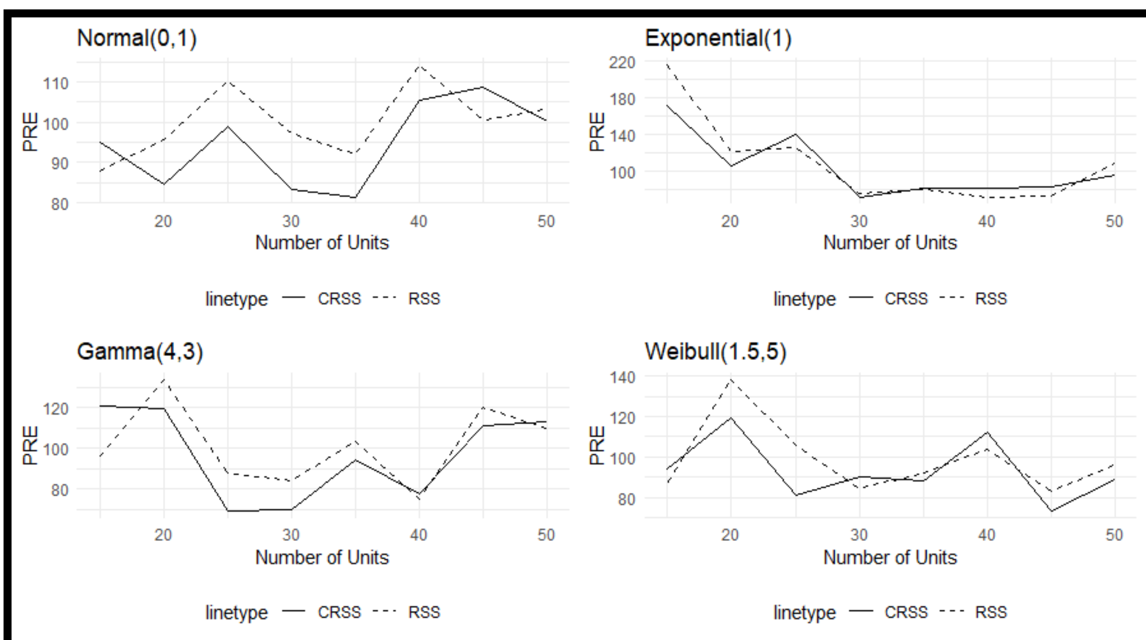

Figure 2: ***PRE comparison for RSS and CRSS***

Under the exponential distribution, the PREs start high for both schemes and decline as units increase. RSS gives an early peak, but CRSS is competitive at larger sizes. Under the Gamma distribution, RSS exhibits sharper variations and an early peak, while CRSS gives smoother and more stable PRE values, outperforming RSS for large unit sizes. Under the Weibull distribution, fluctuations were shown by both schemes, but CRSS became efficient for larger unit sizes and outperformed RSS beyond 40 units. Overall, the general indication of the results was that although RSS sometimes reached a higher initial peak, CRSS showed more stable and robust efficiency across all distributions, especially when the sample size is large and the distribution is highly skewed.

4.2 Illustrative application

In tyre manufacturing, the Banbury mixer is a critical machine in the rubber compounding stage, where temperature control directly affects product quality. In this study, the mixer is monitored over a 12-hour shift 6 a.m. to 6 p.m., totaling 720 minutes. Each overheating event [temperature > 130°C] is recorded, and the time between consecutive events is treated as the inter-arrival time. These times are assumed to follow an exponential distribution. Under stable conditions, the rate parameter is $\lambda = 0.2$, giving an average inter-arrival time of 5 minutes. Over 720 minutes, this produces approximately 140 observations, which are divided into 28 subgroups with set size $m = 5$ for all sampling methods. Three sampling schemes SRS, RSS, and CRSS are applied. The DEWMA control chart is constructed using a smoothing parameter of 0.2 and control limit coefficient $L = 3$.

As the process deteriorates, the rate increases ($\lambda_1 > 0.2$), which shows that the overheating is more frequent. The results demonstrated that all approaches are stable when in control, with the CRSS-DEWMA control chart identifying process shifts earlier compared to RSS-DEWMA and SRS-DEWMA due to its minimized variability and higher efficiency, making it effective for implementing Banbury mixer real-time monitoring. As the process deteriorates, the rate increases ($\lambda_1 > 0.2$), which shows that the overheating is more frequent.

The results demonstrated that all approaches are stable when in control, with the CRSS-DEWMA control chart identifying process shifts earlier compared to RSS-DEWMA and SRS-DEWMA due to its minimized variability and higher efficiency, making it effective for implementing Banbury mixer real-time monitoring. This confirms the superior performance of CRSS and establishes it as the most efficient sampling scheme among those considered.

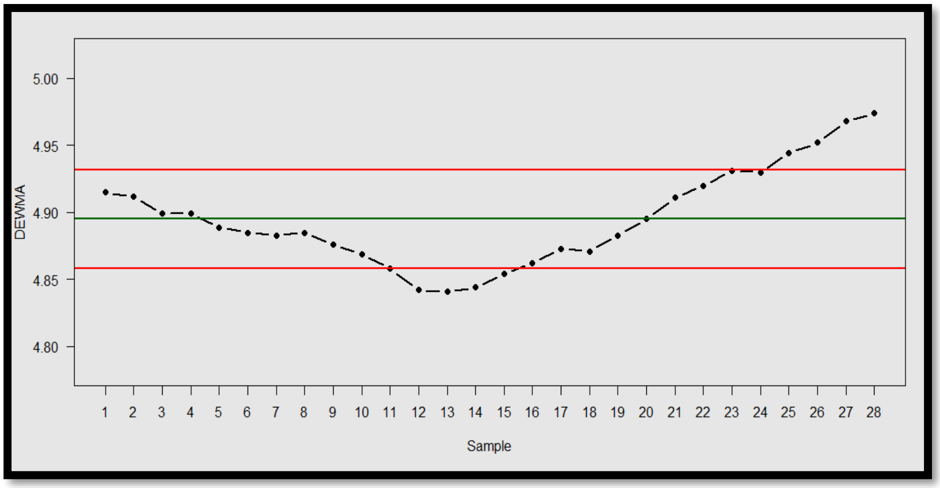

Figure 3: ***DEWMA control chart – SRS***

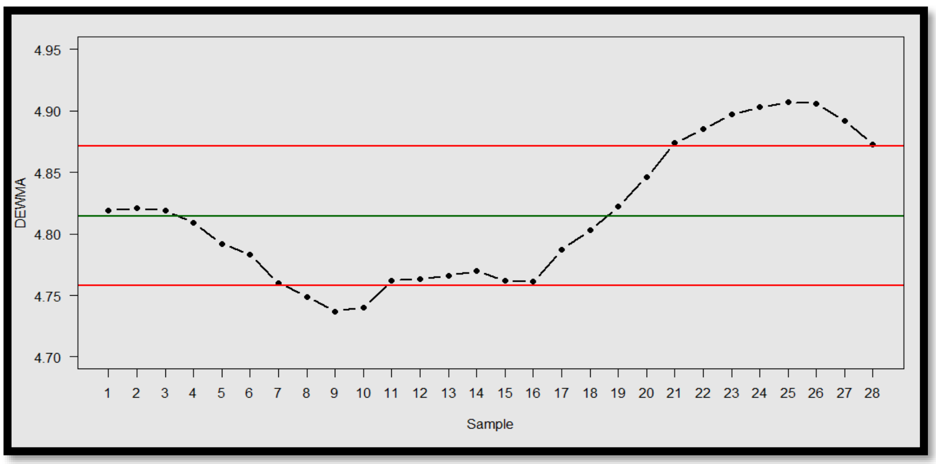

Figure 4: ***DEWMA control chart – RSS***

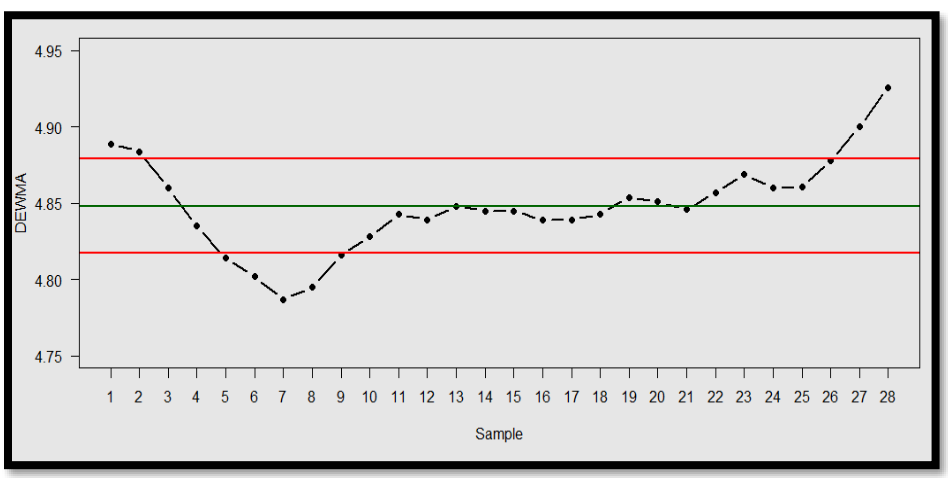

Figure 5: ***DEWMA control chart –CRSS***

5. Concluding remarks

The DEWMA control charts under SRS, RSS, and CRSS sampling schemes show clear differences in monitoring performance. The SRS-based DEWMA chart exhibits higher variability, with four points exceeding the upper control limit, indicating less stable monitoring. The RSS-based DEWMA chart shows improved smoothness but still signals a shift, with five points crossing the upper control limit in the later samples. In contrast, the CRSS-based DEWMA chart displays the most stable behavior, with only two out-of-control points near the end of the sequence and the remaining statistics tightly clustered around the center line. Overall, the CRSS–DEWMA scheme produces fewer out-of-control signals and reduced variability, making it more effective for reliable detection of process changes in the Banbury mixing process.

Acknowledgements

The authors would like to thank the Editors, Associate Editor and Referees for their Suggestions and comments that led to a significant improvement of this manuscript. The author gratefully acknowledges the financial support provided by the University Research Fellowship (URF).

Reference:

1. Patil, G. P., Sinha, A. K., & Taillie, C. (1994). 5 Ranked set sampling. *Handbook of Statistics*, 12, 167–200.
2. Chen, Z., Bai, Z., & Sinha, B. K. (2004). Ranked set sampling: theory and …
3. Stokes, L. (1977). Ranked set sampling with concomitant variables. *Communications in Statistics Theory and Methods*, 6(12), 1207–1211.
4. Patil, G. P., Sinha, A. K., & Taillie, C. (1994). Ranked set sampling. *Handbook of Statistics*, 12, 167–200.
5. Patil, G. P., Sinha, A. K., & Taillie, C. (1999). Ranked set sampling: a bibliography. *Environmental and Ecological Statistics*, 6, 91–98.
6. Chen, Z. (1999). Density estimation using ranked-set sampling data. *Environmental and Ecological Statistics*, 6, 135–146.
7. Chen, Z. (2000). On ranked-set sample quantiles and their applications. *Journal of Statistical Planning and Inference*, 83(1), 125–135.
8. Al-Odat, M. T., & Al-Saleh, M. F. (2001). A variation of ranked set sampling. *Journal of Applied Statistical Science*, 10(2), 137–146.
9. Hossain, S. S., & Muttlak, H. A. (2001). Theory & Methods: Selected ranked set sampling. *Australian & New Zealand Journal of Statistics*, 43(3), 311–325.
10. Nahhas, R. W., Wolfe, D. A., & Chen, H. (2002). Ranked set sampling: cost and optimal set size. *Biometrics*, 58(4), 964–971.
11. Bai, Z., & Chen, Z. (2003). On the theory of ranked-set sampling and its ramifications. *Journal of Statistical Planning and Inference*, 109(1–2), 81–99.

12. Muttlak, H., & Al-Sabah, W. (2003). Statistical quality control based on ranked set sampling. *Journal of Applied Statistics*, 30(9), 1055–1078.
13. Chen, Z., Bai, Z., & Sinha, B. K. (2004). Ranked set sampling: theory and applications (Vol. 176). Springer.
14. Wolfe, D. A. (2004). Ranked set sampling: an approach to more efficient data collection.
15. Sinha, A. K. (2005). On some recent developments in ranked set sampling.
16. Patil, G. P. (2006). Ranked set sampling. *Encyclopedia of Environmetrics*.
17. Balakrishnan, N., & Li, T. (2008). Ordered ranked set samples and applications to inference. *Journal of Statistical Planning and Inference*, 138(11), 3512–3524.
18. Kadilar, C., Unyazici, Y., &Cingi, H. (2009). Ratio estimator for the population mean using ranked set sampling. *Statistical Papers*, 50, 301–309.
19. Wolfe, D. A. (2010). Ranked set sampling. *Wiley Interdisciplinary Reviews: Computational Statistics*, 2(4), 460–466.
20. Wolfe, D. A. (2012). Ranked set sampling: its relevance and impact on statistical inference. *International Scholarly Research Notices*, 2012(1), 568385.
21. Gemayel, N. M., Stasny, E. A., Tackett, J. A., & Wolfe, D. A. (2012). Ranked set sampling: an auditing application. *Review of Quantitative Finance and Accounting*, 39, 413–422.
22. Haq, A., Brown, J., Moltchanova, E., & Al-Omari, A. I. (2013). Partial ranked set sampling design. *Environmetrics*, 24(3), 201–207.
23. Haq, A., Brown, J., Moltchanova, E., & Al-Omari, A. I. (2014). Mixed ranked set sampling design. *Journal of Applied Statistics*, 41(10), 2141–2156.
24. Zamanzade, E., & Al-Omari, A. I. (2016). New ranked set sampling for estimating the population mean and variance. *Hacettepe Journal of Mathematics and Statistics*, 45(6), 1891–1905.
25. Wang, X., Lim, J., & Stokes, L. (2016). Using ranked set sampling with cluster randomized designs. *Journal of the American Statistical Association*, 111(516), 1576–1590.
26. Bouza-Herrera, C. N., & Al-Omari, A. I. F. (2018). Ranked set sampling: 65 years of improving the accuracy in data gathering. Academic Press.
27. Raqab, M. Z., &Qiu, G. (2019). On the extropy properties of ranked set sampling. *Statistics*, 53(1), 210–226.
28. Taconeli, C. A., & Bonat, W. H. (2020). On the performance of estimation methods under ranked set sampling. *Computational Statistics*, 35(4), 1805–1826.
29. Mohammadkhani, A., Amiri, A., & Khoo, M. B. (2023). A review of ranked set sampling and modified methods in designing control charts. *Quality and Reliability Engineering International*, 39(4), 1465–1493.