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Comparing Quantum Harmonic Oscillator Model with Other Financial Models to Price European Call Options

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Abstract: An option is a financial contract that gives holder the right, but not the obligation, to buy or sell an underlying asset at the strike price on or before the expiration date.Options are significant financial instruments that offer investors and traders a variety of risk management tools and trading strategies.There are many financial models that can be usedfor options pricinglike Black Scholes option pricing model, Heston Stochastic Volatility Model, Vasicek Mean Reversion Model etc. This paper proposes Quantum harmonic oscillator model based on quantum mechanics for pricing European call options and will compare its estimated value with the actual price and the pricing obtained by other models.
Keywords: Quantum harmonic oscillator, Option Pricing, Black Scholes option pricing model, Heston Stochastic Volatility Model, Vasicek Mean Reversion Model.

Introduction:

This paper discusses about different approaches to price the options. It is a fundamental concept in finance that has been widely studied and applied in both academia and industry. Options are contracts that give the holder the right, but not the obligation, to buy or sell an underlying asset at a predetermined price and time. The value of an option is determined by a variety of factors, including the current price of the underlying asset, the time until the option expires, the volatility of the underlying asset, and prevailing interest rates.

The pricing of options is a complex process that has been the subject of extensive research in financial economics. One of the most widely used models for option pricing is the Black-Scholes model. The model assumes that the underlying asset follows a log-normal distribution, and it considers the current price of the underlying asset, the option's strike price, the time until the option expires, the volatility of the underlying asset, and the prevailing interest rates.

The Black-Scholes model has been widely studied and criticized, leading to the development of alternative models, such as the binomial model, which considers multiple possible outcomes for the price of the underlying asset of the options at the expiry. Other models, such as the Monte Carlo simulation, Vasicek Mean Reversion Model, and the Heston Stochastic volatility model, have also been developed to price options.

Option pricing has important implications for traders and investors, as it helps them determine the fair value of an option and make informed decisions about buying, selling, or holding options. In addition, option pricing has practical applications in risk management, portfolio optimization, and financial engineering but financial models have their own limitations which restricts them to reach to the higher accuracy levels. So, it is important in finance to use different concepts to validate accuracy.

To overcome these limitations of the above-mentioned financial models, this paper introduces Quantum Harmonic oscillator model to price European call options which is a theoretical framework used to describe the behaviour of certain physical systems, such as the vibrations of a molecule or the motion of a particle in a potential well. In this model, the energy of the system is quantizedi.e.,it can only take on certain discrete values.Moreover, the quantum harmonic oscillator model is particularly useful in option pricing because it provides a way to incorporate the effects of market volatility, which is a key factor in determining the price of options.It is a model with closed-form solution, which means that there is a mathematical formula to compute the volatility and price of the options.

In financial options pricing models, volatility refers to the expected standard deviation of the underlying asset's price over a certain period of time. On the other hand, in quantum harmonic oscillator, volatility refers to the standard deviation of the position of the particle in the harmonic potential well.The standard deviation of position in the harmonic potential well changes as the particle's energy level changes. So, in quantum harmonic oscillator, volatility is not constant and changes as the energy level changes.The QHO model has been used as an alternative to the Black-Scholes model for option pricing, as it can capture some of the non-linear and non-normal characteristics of stock price movements that the Black-Scholes model cannot. However, it has its own limitations and should be used with caution, especially for more complex financial instruments.

To validate the QHO model, it is important to compare its predictions with market prices and other models, as well as to conduct sensitivity analysis on the input parameters, such as volatility and time to expiration. This paper introduces some more different option pricing models including black scholes model to price European call options to compare it with the same derived from quantum harmonic oscillator.

Here, themain difference in volatility among these option pricing models is mentioned below.It is the way in which individual model is constructed and this volatility impacts on the prediction of option pricing.

- Quantum Harmonic Oscillator:
  Volatility is not explicitly modelled as a separate input parameter. Instead, it is indirectly captured by the drift and diffusion terms of the stochastic process that describes the evolution of the underlying asset price.

- Black Scholes Option Pricing Model:
  In this model, volatility is assumed to be constant over time and is an input parameter in the formula. It represents the degree of fluctuation of the underlying asset's price and is a measure of the risk associated with the option.

- Heston Stochastic Volatility Model:
  Volatility is modelled as a stochastic process that follows a mean-reverting square root process. It is assumed to be a function of time and the underlying asset's price. It plays a central role in the option pricing formula.

- Vasicek Mean Reversion Model:
  It is an input parameter that represents the degree of randomness in the interest rate process and affects the option prices via the term structure of interest rates. So, volatility is modelled as a mean-reverting process that follows an Ornstein-Uhlenbeck process.

- Monte Carlo Option Pricing Model:
  It is simulated using random number generators, and its value at each time step affects the simulation of the underlying asset's price. In this model, volatility is developed as a stochastic process that is a function of time and the underlying asset's price. The option price is then estimated by averaging the payoffs of a large number of simulated paths.

Literature Review:

Ivancevic [8] (2009) proposed a bidirectional quantum associative memory structure for Black–Scholes–like option price progression. It comprised of a pair of coupled NLS equations, one controlling stochastic volatility as well as the other administering option price, both self-organizing in an adaptive 'market heat potential' trained by continuous Hebbian learning. By using approach of lines with adaptive step-size integrator, this stiff pair of NLS equations were solved numerically. He established a quantum neural composition approach for option price modelling.

Alvarez [2] (2010) used lagged detrended fluctuation analysis (DFA) to investigate the effectiveness of crude oil markets (i.e., autocorrelations are dependent of the time scale). Results using spot price data for the years 1986 to 2009 showed significant efficiency deviations linked to lag autocorrelations, therefore implementing the random walk for crude oil prices entails significant forecasting costs.

In't Hout [7] (2010) dealt with the numerical solution of the Heston partial differential equation (PDE) that plays an important role in financial option pricing theory, Heston (1993). A feature of this time-dependent, twodimensional convection-diffusion-reaction equation is the presence of a mixed spatial-derivative term, which stems from the correlation between the two underlying stochastic processes for the asset price and its variance.

Cotfas&Cotfas[5](2013) investigates from a mathematical point of view an extension directly related to the quantumharmonic oscillator. In the considered case, the solution is the sum of aseries involving the Hermite-Gauss functions. A finite-dimensional version isobtained by using a finite oscillator and the Harper functions. This simplifiedmodel keeps the essential characteristics of the continuous one and uses finitesums instead of series and integrals.

Garrahan[6] (2018)examined the ability of GBM, a modified GBM with an Ornstein-Uhlenbeck mean reversion term, and the QHO to model the behaviours of a return distribution over time. The models were applied to S&P 500 returns over the last five years. For each model, parameters were chosen by minimizing a goodness of fitstatistic using parameter search algorithms. The optimal results for each model were compared,using the Cramer von Mises test statistic for multiple return windows.

Ahn[1] et al. (2018) developed a quantum harmonic oscillator as a model for the market force that pulls a stock return from short-run oscillations to the long-run equilibrium. Additionally, using analogies, they established an economic justification for physics notions like the eigenstate, eigenenergy, and angular frequency, which clarifies the connection between the literature on finance and econophysics.

Jeknić-Dugić[9] (2018) pursued the quantum-mechanical challenge to the efficient market hypothesis for the stock market by employing the quantum Brownian motion model. He also introduced the external harmonic field for the Brownian particle and use the quantum Caldeira-Leggett master equation as a potential phenomenological model for the stock market price fluctuations.

Lee [11] et al. (2020) examined the weak-form efficient market hypothesis of the crude palm oil market by adopting the quantum harmonic oscillator. This approach allows Lee to analyze market efficiency by estimating one parameter: the probability of finding the market in a ground state where conclusion confirmed that the crude palm oil market is more efficient than the West Texas Intermediate crude oil market.

Orrell [13] (2020) addressed issues regarding intrinsically uncertain demand by using a quantum framework to model supply and demand as, not a cross, but a probabilistic wave, with an associated entropic force. The approach is used to derive from first principles a technique for modelling asset price changes using a quantum harmonic oscillator, that has been previously used and empirically tested in quantum finance. The method is demonstrated for a simple system, and applications in other areas of economics are discussed.

Samimi& Najafi [15] (2021) studies the European option pricing on the zero-coupon bond in which the Skew Vasicek model uses to predict the interest rate amount. To do this, he applied the skew Brownian motion as the random part of the model and showed that results of the model predictions are better than other types of the model.

Bhatt and Gor[3] (2022) showcased an interesting structure of Risk Neutral system. They also examine single step and multistep quantum binomial option pricing model. This approach elaborates circuit proposed by A. Meyer. Bhatt and Gor[3] (2022) review applications of quantum harmonic oscillator model in financial mathematics and also discussed about different applications of quantum harmonic oscillator and its characteristics.

Data Collection:

In this paper, historical data is collected randomly from YAHOO Finance website for options of Silver-gate Capital Corporation (SI). Historical data for the same is collected in the time span of one year from 1st January 2021 to 1st January 2022.

Methodology:

Quantum Harmonic Oscillator

This paper uses Quantum Harmonic Oscillator (QHO) model to price a call option of a stock. It implements the QHO model using the Crank-Nicolson method with a backward propagation in time.The option is priced by propagating the wave function of the underlying stock price backward in time from the expiration date to the current date. The QHO potential is used to model the stock price dynamics, with the drift and diffusion terms defined in terms of the risk-free interest rate, volatility, and the frequency of the QHO potential.

Here for Silver-gate Capital Corporation, the calculations were made with python programming. Below mentioned algorithms were developed in this paper to price the European call options using quantum harmonic oscillator.

1. Set up the initial parameters:
   Current stock price (So), Strike price (K), Risk-free interest rate (r), Volatility (sigma), Time to expiration (T), Number of time steps (N), and Number of simulations (M).

2. Set up the grid for the wave function:
   Number of grid points (N_{grid}), Minimum value of x-axis (x_{min}), Maximum value of x-axis (x_{max}) and spacing between two consecutive grid points (dx).

3. Set up the potential for the QHO using the formula:
$$V = \frac{1}{2}(\omega^2 x^2)$$
   where $\omega = \sqrt{2r\sigma^2}$

4. Define the drift (μ) and diffusion (D) terms for the stochastic process using the formulas:
$$\mu = r - \frac{1}{2}\sigma^2$$
$$D = \frac{\sigma}{\sqrt{2\omega}}$$

5. Initialize the wave function at expiration using the formula:
$$\varphi = \sqrt{(max(S_0 - k, 0))} * exp\left(-\frac{1}{2}\frac{\left(x - log\left(\frac{S_0}{k}\right)\right)^2}{D^2}\right)$$
   and
$$|\varphi| = \sqrt{\left(\int \varphi^2 \, dx\right)}$$

6. Propagating the wave function backwards in time using the following steps:
   a. Calculate the new potential and drift terms using the formula:

$$V_{new} = V + \frac{1}{2}\omega^2(x - \mu dt)^2$$

$$\mu_{new} = \mu - \frac{1}{2}\frac{\sigma^2}{\omega} + \frac{1}{2}\frac{jD^2(x - \mu dt)}{\omega^2}$$

where $dt = \frac{T}{N}$is the time step.

b. Construct the matrix for the evolution operator using the formula:

$$\hat{U} = \begin{pmatrix} e^{\frac{iE_0 t}{\hbar}} & 0 & \cdots & 0 \\ \vdots & e^{\frac{iE_1 t}{\hbar}} & \ddots & \vdots \\ 0 & 0 & \cdots & e^{\frac{iE_{n-1} t}{\hbar}} \end{pmatrix}_{(n-1)\times(n-1)}$$

where $(n-1)$= grid

c. Apply the evolution operator to the wave function.

7. Calculate the expected payoff using the formula:

$$payoff = max(exp(x) - k, 0)$$

8. Calculate the call option price by taking the dot product of the squared wave function with the payoff and discounting it to the present value using the formula:

$$Call\ Price = (\varphi^2 \cdot payoff) * exp(-rT)$$

Black Scholes Option Pricing Model

The Black-Scholes model is a mathematical formula used to calculate the theoretical value of a European call option. The formula considers various factors that affect the price of the option, such as the current stock price, the exercise price, the time until expiration, the risk-free interest rate, and the volatility of the underlying asset. Here is the formula for the Black-Scholes model for a European call option:

$$C = SN(d_1) - xe^{-rT}N(d_2)$$

where,

C = The theoretical value of the call option

S = The current stock price

X = The exercise price of the option

r = The risk-free interest rate

T = The time to expiration of the option (in years)

N() = The cumulative standard normal distribution

$$d_1 = \frac{\left[\ln\left(\frac{S}{X}\right) + \left(r + \frac{(\sigma^2)}{2}\right) * T\right]}{\sigma * \text{sqrtT}}$$

$$d_2 = d_1 - \sigma * \text{sqrt}(T)$$

In this formula, σ represents the volatility of the underlying asset, which is a measure of how much the price of the asset fluctuates over time. The d_1 and d_2 terms are known as the "Black-Scholes parameters" and are used to calculate the probability that the option will be exercised. The $N(d_1)$ and $N(d_2)$ terms represent the cumulative standard normal distribution, which is a statistical measure that calculates the probability of a particular event occurring.

By plugging in the relevant values for S, X, r, T, and σ into the Black-Scholes formula, one can calculate the theoretical value of a European call option. The formula assumes that the underlying asset follows a log-normal distribution and that there are no transaction costs or taxes. However, in practice, these assumptions may not hold, and adjustments may need to be made to the formula to account for these factors.

Heston Stochastic Volatility Model:

The Heston model assumes that the underlying asset follows a geometric Brownian motion process, which is similar to the Black-Scholes model. However, the model also incorporates stochastic volatility, which means that the volatility of the underlying asset is not constant but rather fluctuates over time.

Here is the formula for the Heston stochastic volatility model for a European call option:

$$C = SP_1 - Xe^{-rT} * P_2$$

where,

C = The theoretical value of the call option

S = The current stock price

X = The exercise price of the option

r = The risk-free interest rate

T = The time to expiration of the option (in years)

P_1 = The probability density function for the stock price, under the risk-neutral measure, at time T

P_2 = The probability density function for the log stock price, under the risk-neutral measure, at time T

The probability density functions P_1 and P_2 are calculated using the Heston model's stochastic volatility process, which is represented by two stochastic differential equations:

$$dS = rSdt + \sqrt{v}SdW_1 \; dv = \kappa(\theta - v)dt + \sigma\sqrt{v} * dW_2$$

Here,
dS = The change in the stock price
v = The volatility of the underlying asset
κ = The mean-reverting rate of the volatility process
θ = The long-term average volatility
σ = The volatility of the volatility process
dW_1 anddW_2= Two Wiener processes that represent the randomness in the stock price and volatility processes, respectively.

By numerically solving these stochastic differential equations, one can simulate the paths of the stock price and volatility processes, and then use them to calculate the probability density functions P_1 and P_2. Once these probability density functions are known, one can use them to calculate the theoretical value of a European call option using the above formula.

The Heston model is more complex than the Black-Scholes model, but it is considered to be a more accurate representation of the real-world behaviour of financial assets, as it accounts for changes in volatility over time. However, the Heston model can also be computationally intensive.

Vasicek Mean Reversion Model

The Vasicek model assumes that the short-term interest rate follows a mean-reverting process, as described by the following stochastic differential equation:

$$dr(t) = k\big(\theta - r(t)\big)dt + \sigma\big(r(t)\big)dW(t)$$

where,
dr(t) = the change in the short-term interest rate at time t
k = the mean-reversion rate of the interest rate process
θ = the long-term average interest rate
σ(r(t)) = the volatility of the interest rate at time t, which is a function of the interest rate r(t)
dW(t) = a Wiener process representing the randomness in the interest rate process

Using this model, the price of a European call option can be calculated using the following formula:

$$C = SN(d_1) - xe^{-rT}N(d_2)$$

where all other parameters are as same as in the above-mentioned models.

$$d1 = \frac{\left(\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T\right)}{\sigma\sqrt{T}}$$

$$d2 = d1 - \sigma\sqrt{T}$$

To use the Vasicek model in this formula, one can replace the risk-free interest rate with the expected value of the short-term interest rate under the Vasicek model:

$$r = E(r) = \theta + (r_0 - \theta) * e^{-kT}$$

Where: r_0 = The current short-term interest rate

Once the expected value of the short-term interest rate is known, it can be used to calculate the price of the European call option using the above formula.

Monte Carlo Option Pricing Model

The Monte Carlo option pricing model is a numerical method for estimating the price of a European call option. It involves simulating multiple possible future stock price paths using random numbers and then using these paths to calculate the expected payoff of the option.

Steps to showcases Monte Carlo Option Pricing Model:

1. Generate a large number of random numbers, typically using a pseudorandom number generator. These numbers will be used to simulate possible future stock price paths.
2. Calculate the drift and volatility of the stock price using historical data and a mathematical model such as the Black-Scholes model. These values will be used to simulate the future stock price paths.
3. Set an initial stock price and calculate the time step size, which is the length of time between each simulated stock price.
4. Simulate multiple future stock price paths by randomly selecting from the generated numbers to create a sequence of price changes over time.
5. For each simulated stock price path, calculate the payoff of the European call option at the time of expiration using the formula:

   $$\text{Payoff} = \max(0, S(T) - X)$$

   Where S(T) = the simulated stock price at the time of expiration and X = the strike price of the option

6. Repeat steps 4-5 for a large number of simulated stock price paths.
7. Calculate the average payoff of the European call option across all the simulated stock price paths. This average value represents the estimated price of the option.

The Monte Carlo option pricing model is flexible and can be used to price options with complex payoff structures, as well as options on assets with non-normal return distributions.

Numerical Calculations:

Descriptive analysis for the data of Silver Gate Capital Corporation is done with the help of JAMOVI.

Table 1 Descriptive Analysis

Descriptives	
Log Return	
Total observations	18
Strike Price	2.5
Current price	12.8
risk free rate	3%
N	17
Mean	0.0195
Median	-0.0245
Mode	-0.116
Standard deviation	0.118
Variance	0.014
Shapiro-Wilk W	0.871
Shapiro-Wilk p	0.023

Quantum Harmonic Oscillator

Parameters used in quantum harmonic oscillator and option pricing with the help of model is mentioned below:

Table 2 Quantum Harmonic Oscillator

Parameters		QHO Option Pricing Model	
Planck constant	6.26E-34	Payoff	7.94E+00
Diffusion coefficient	0.00019	Call Value	10.44
Mass	1.03E-63		
k	0.0001	Absolute (d1)	0.23683
w	3.12E+29	Absolute (d2)	4.04E-01
mw	3.21E-34		
mean square	0.00038	N(d1)	0.593606
Gamma	9.75E-05	N(d2)	6.57E-01

Black Scholes Option Pricing Model

Payoff and the call option value predicted by the Black Scholes Option pricing model is calculated as:

Table 3 Black-Scholes Model

BS Option Pricing Model	
Payoff	7.29973
Call Value	9.79973
Absolute (d1)	1.656365
Absolute (d2)	1.64763
N(d1)	0.951176
N(d2)	0.950286

Heston Stochastic Volatility Model

Parameters used in Heston Stochastic Volatility Model are calculated with MS Excel. It uses some different parameters such as theta, kappa sigma described below:

Table 4 Heston Model

Heston Stochastic Volatility Model	
Theta	0.0195
Kappa	3.5
Sigma	0.118
Rho	-0.5
v_0	0.013924
Payoff	8.227
Call Value	10.727

Vasicek Mean Reversion Model

Vasicek model uses many of the parameters from Heston model with slight modifications mentioned below:

Table 5Vasicek Model

Vasicek Model	
r_0	0.03
K	0.1
Theta	0.05
Sigma	0.05
T	1
Dt	0.00274
N	1000
Payoff	7.6157
Call Value	10.1157

Monte Carlo Option Pricing Model

This is a simulation method to predict option pricing, so it uses limited parameters for prediction calculated in the given table:

Table 6Monte Carlo Model

Monte Carlo Model	
S_0	12.8
k	2.5
r	0.03
T	1
sigma	0.118
N	100000
dt	0.003968
Payoff	6.6
Call Value	9.1

Graphical Representation:

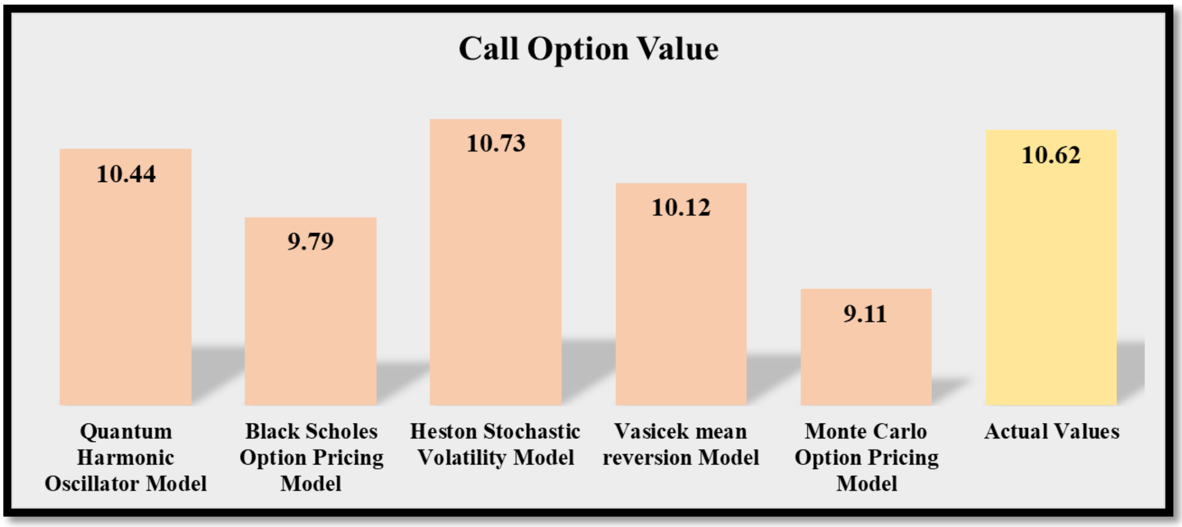

Figure 1 Graphical Representation

It is important to mention here that the actual price of Silver Gate Capital Corporation at expiration date is 10.62 INR and predicted value of Heston model and quantum harmonic oscillator model is 10.73 INR and 10.44 INR respectively which can be observed from the graph given above.

Conclusion:

This study compared effectiveness ofquantum harmonic oscillator with the four different models in predicting European call option prices: Black-Scholes, Vasicek, Monte Carlo, and Heston. It is observed that the Heston model provided the best fit to historical option prices, followed closely by the quantum harmonic oscillator model. While the Monte Carlo method showed promise in its ability to handle complex option structures, it did not perform well as the other two models in this study.

These results highlight the importance of considering alternative models beyond the traditional Black-Scholes framework for option pricing. This study suggests that the Heston and quantum harmonic oscillator models may provide better predictions in this particular case where the actual price of Silver Gate Capital Corporation at expiration date is 10.62 INR which is very much closure to the predicted value of Heston and Quantum Harmonic Oscillator. While other three models underperform for the situation.

Moreover, it is found that the quantum harmonic oscillator model provided a surprisingly accurate fit to historical option prices, especially considering it as a relatively simple structure. This model is based on the idea that stock prices can be treated as wave functions and can be described using principals of quantum mechanics.

While the quantum harmonic oscillator model is not widely used in finance and economics, our results suggest that it may have potential applications in option pricing and other financial modelling contexts.

Future Scope:
This study found that Quantum Harmonic Oscillator model provided a surprisingly accurate fit to historical option prices, and future research could explore its theoretical underpinnings and potential applications in other financial modelling contexts.This paper was based on historical data from a single market, and future research could explore how the models perform in different market conditions or with different types of options.There are many other factors that can influence option prices, such as interest rates, dividends, and volatility. In Future, one could explore how to incorporate these additional factors into the models. Future research could also compare the models with additional benchmarks such as implied volatility or out-of-sample prediction accuracy. In future another approach can be developed through the techniques such as bootstrapping or cross-validation to estimate the uncertainty in the parameters of Quantum Harmonic Oscillator and their impact on the option prices. This would provide more realistic assessment and performance of the models in the practiceand help to identify any potential weaknesses or limitations of the models.

References:

1. Ahn, K., Choi, M., Dai, B., Sohn, S., & Yang, B. (2018). Modeling stock return distributions with a quantum harmonic oscillator. EPL (Europhysics Letters), 120(3), 38003.
2. Alvarez-Ramirez, J., Alvarez, J., & Solis, R. (2010). Crude oil market efficiency and modeling: Insights from the multi-scaling autocorrelation pattern. Energy Economics, 32(5), 993-1000.
3. Bhatt A., Gor R. (2022). Examining Single step and Multistep Quantum Binomial Option Pricing Model. IOSR Journal of Economics and Finance (IOSR-JEF) e-ISSN: 2321-5933, p-ISSN: 2321-5925.Volume 13, Issue 3 Ser. IV (May. – June. 2022), PP 52-60 www.iosrjournals.org
4. Bhatt A., Gor R. (2022). A Review paper on Quantum harmonic oscillator and its applications in finance. IOSR Journal of Economics and Finance (IOSR-JEF) e-ISSN: 2321-5933, p-ISSN: 2321-5925.Volume 13, Issue 3 Ser. IV (May. – June. 2022), PP 61-65 www.iosrjournals.org
5. Cotfas, L. A., &Cotfas, N. (2013). Quantum harmonic oscillator in option pricing. arXiv preprint arXiv:1310.4142.

6. In't Hout, K. J., & Foulon, S. (2010). ADI finite difference schemes for option pricing in the Heston model with correlation. International Journal of Numerical Analysis &Modeling, 7(2).
7. Ivancevic, V. G. (2009). Quantum Neural Computation for Option Price Modelling. arXiv preprint arXiv:0903.0680.
8. Jeknić-Dugić, J., Petrović, I., Arsenijević, M., &Dugić, M. (2018). Quantum Brownian oscillator for the stock market. arXiv preprint arXiv:1901.10544.
9. Kienzler, D., Lo, H. Y., Keitch, B., De Clercq, L., Leupold, F., Lindenfelser, F., ... & Home, J. P. (2015). Quantum harmonic oscillator state synthesis by reservoir engineering. Science, 347(6217), 53-56.
10. Lee, G. H., Joo, K., &Ahn, K. (2020, July). Market efficiency of the crude palm oil: Evidence from quantum harmonic oscillator. In Journal of Physics: Conference Series (Vol. 1593, No. 1, p. 012037). IOP Publishing.
11. Podlozhnyuk, V., & Harris, M. (2008). Monte carlo option pricing. CUDA SDK.
12. Orrell, D. (2020). A quantum model of supply and demand. Physica A: statistical Mechanics and its Applications, 539, 122928.
13. Orrell, D. (2021). A quantum oscillator model of stock markets. Available at SSRN 3941518.
14. Samimi, H., & Najafi, A. (2021). Bond and option prices under skew vasicek model with transaction cost. Mathematical Problems in Engineering, 2021, 1-8.
15. Um, C. I., Yeon, K. H., & George, T. F. (2002). The quantum damped harmonic oscillator. Physics Reports, 362(2-3), 63-192.