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Revan Indices of Certain Graph Models

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Abstract: There are many topological indices. Among the degree based topological indices, Randic index Zagreb indices, Banhatti indices etc. The Revan vertex degree of a vertex in G is defined as $r_G(v)=\Delta(G)+\delta(G)-d_G(v)$. The revan edge connecting the revan vertices u and v will be denoted by uv. The first and second Revan indices of a graph G, defined as $R_1(G)=\sum_{uv\in E(G)}|r_G(u)+r_G(v)|$ and $R_2(G)=\sum_{uv\in E(G)} r_G(u)r_G(v)$. In this paper we obtain the first and second Revan indices of certain graphs say square path, square cycle, wheel graph, fan graph and comb graph.

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Key words: Revan Indices, square path square cycle, wheel graph and fan graph.

1. Introduction

Let G be a finite, simple connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree of a vertex v is the number of vertices adjacent to v. $\Delta(G)$ and $\delta(G)$ denote the maximum and minimum degree among the vertices of G. We refer [1] for undefined term and notation. A topological index is a numerical parameter mathematically derived from the graph structure. Numerous such topological indices have been considered in theoretical chemistry and have some applications. The Revan vertex degree of a vertex in G is defined as $r_G(v)=\Delta(G)+\delta(G)-d_G(v)$. The Revan edge connecting the Revan vertices u and v will be denoted by uv. The first and second Revan indices of a graph G, defined as $R_1(G)=\sum_{uv\in E(G)}|r_G(u)+r_G(v)|$ and $R_2(G)=\sum_{uv\in E(G)} r_G(u)r_G(v)$.

2. Preliminaries

Definition 2.1 The square of a graph G is obtained by starting with G, and adding the edges between two vertices whose distance in G is two.

Definition 2.2 For a graph, the maximum degree denoted by $\Delta(G)$, is the vertex with greatest number of edges incident to it. The minimum degree denoted by $\delta(G)$, is the degree of the vertex with least number of edges incident to it.

Definition 2.3 The Revan vertex degree of a vertex in G is defined as $r_G(v)=\Delta(G)+\delta(G)-d_G(v)$.

Definition 2.4 The first and second Revan indices of a graph *G*, defined as $R_1(G)=\sum_{uv\in E(G)}\left|r_G(u)+r_G(v)\right|$ and $R_2(G)=\sum_{uv\in E(G)} r_G(u)r_G(v)$.

Definition 2.5 Comb is a graph obtained by joining a single pendent edge to each vertex of a path.

Definition 2.6 The corona graph $G_1 * G_2$ of two graphs G_1 and G_2 is graph G obtaining by taking one copy o of fG_1 which has p_1- vertices ,and p_1 copies of G_2 and then joining i^{th} vertex G_1 to every vertex in the i^{th} copy of G_2.

3. Main Results

Theorem 3.1 Let P_n is the path graph then

1. $R_1(P_n)=2n$
2. $R_2(G)=n+1$

Proof. Let G be the graph P_n. In the path graph by the algebraic method there are two types of edges on the degree of end vertices as follows

$$E_{12}=\{uv\in E(G)/d_G(u)=1\,\&\,d_G(v)=2\},\ |E_{12}|=2$$

$$E_{22}=\{uv\in E(G)/d_G(u)=2\,\&\,d_G(v)=2\},\ |E_{22}|=n-3.$$

Thus we have two types of revan edges based on the degree of the end revan vertices of each edge as follows, we have $\Delta(G)+\delta(G)=3$.

$$RE_{21}=\{uv\in E(G)/r_G(u)=2\,\&\,r_G(v)=1\},\ |RE_{21}|=2$$

$$RE_{11}=\{uv\in E(G)/r_G(u)=1\,\&\,r_G(v)=1\},\ |RE_{11}|=n-3$$

1. To compute $R_1(P_n)$, we see that

$$\begin{aligned}R_1(G) &= \sum_{uv\in E(G)}\left|r_G(u)+r_G(v)\right| \\ &= \sum_{RE_{21}}\left|r_G(u)+r_G(v)\right|+\sum_{RE_{11}}\left|r_G(u)+r_G(v)\right| \\ &= 2(2+1)+(n-3)(1+1) \\ &= 6+2n-6\end{aligned}$$

$$R_1(G)=2n$$

2. To compute $R_2(P_n)$, we see that

$$R_2(G)=\sum_{uv\in E(G)}\left|r_G(u)r_G(v)\right|$$

$$R_2(G)=\sum_{RE_{21}}\left|r_G(u)r_G(v)\right|+\sum_{RE_{11}}\left|r_G(u)r_G(v)\right|$$

$$R_2(G) = \sum_{RE_{21}} |r_G(u) r_G(v)| + \sum_{RE_{11}} |r_G(u) r_G(v)|$$
$$= 2(1\times 2) + (n-3)(1\times 1)$$
$$= 4 + n - 3$$
$$R_2(G) = n+1$$

□

Theorem 3.2 Let $P_n^{\,2}$ is the square path graph then

1. $R_1(P_n^2) = 8n+2$
2. $R_2\left(P_n^2\right) = 8n+20$

Proof. Let G be the graph P_n^2. In the square path graph by the algebraic method there are four types of edges on the degree of end vertices as follows

$$E_{24} = \{uv \in E(G) / d_G(u) = 2 \,\&\, d_G(v) = 4\},\ |E_{24}| = 2$$
$$E_{23} = \{uv \in E(G) / d_G(u) = 2 \,\&\, d_G(v) = 3\},\ |E_{23}| = 2$$
$$E_{34} = \{uv \in E(G) / d_G(u) = 3 \,\&\, d_G(v) = 4\},\ |E_{23}| = 4$$
$$E_{44} = \{uv \in E(G) / d_G(u) = 4 \,\&\, d_G(v) = 4\},\ |E_{44}| = 2n-11$$

Thus we have four types of revan edges based on the degree of the end revan vertices of each edge as follows, we have $\Delta(G) + \delta(G) = 6$.

$$RE_{43} = \{uv \in E(G) / r_G(u) = 4 \,\&\, r_G(v) = 3\},\ |RE_{43}| = 2$$
$$RE_{42} = \{uv \in E(G) / r_G(u) = 4 \,\&\, r_G(v) = 2\},\ |RE_{42}| = 2$$
$$RE_{32} = \{uv \in E(G) / r_G(u) = 3 \,\&\, r_G(v) = 2\},\ |RE_{32}| = 4$$
$$RE_{22} = \{uv \in E(G) / r_G(u) = 2 \,\&\, r_G(v) = 2\},\ |RE_{43}| = 2n-11$$

1. To compute $R_1(P_n^2)$, we see that

$$R_1(G) = \sum_{uv\in E(G)} |r_G(u) + r_G(v)|$$
$$= \sum_{RE_{43}} |r_G(u) + r_G(v)| + \sum_{RE_{42}} |r_G(u) + r_G(v)| + \sum_{RE_{32}} |r_G(u) + r_G(v)| + \sum_{RE_{22}} |r_G(u) + r_G(v)|$$
$$= 2(4+3) + 2(4+2) + 4(3+2) + (2n-11)(2+2)$$
$$= 14 + 12 + 20 + (2n-11)4$$
$$= 46 + 8n - 44$$
$$R_1(G) = 8n+2$$

2. To compute $R_2(P_n^2)$, we see that

$$R_2(G) = \sum_{uv\in E(G)} |r_G(u) r_G(v)|$$

$$R_2(G) = \sum_{RE_{43}} |r_G(u) r_G(v)| + \sum_{RE_{42}} |r_G(u) r_G(v)| + \sum_{RE_{32}} |r_G(u) r_G(v)| + \sum_{RE_{22}} |r_G(u) r_G(v)|$$
$$= 2(4\times3) + 2(4\times2) + 4(3\times2) + (2n-11)(2\times2)$$
$$= 24 + 16 + 24 + 8n - 44$$
$$R_2(G) = 8n + 20$$

□

Theorem 3.3 Let C_n is the cycle graph then

1. $R_1(C_n) = 4n$
2. $R_2(C_n) = 4n$

Proof. Let G be the graph C_n. In the cycle graph by the algebraic method there is one type of edges on the degree of end vertices as follows

$E_{22} = \{uv \in E(G) / d_G(u) = 2 \,\&\, d_G(v) = 2\},\ |E_{22}| = n$

Thus we have one type of revan edges based on the degree of the end revan vertices of each edge as follows, we have $\Delta(G) + \delta(G) = 4$.

$RE_{22} = \{uv \in E(G) / r_G(u) = 2 \,\&\, r_G(v) = 2\},\ |RE_{22}| = n$

1. To compute $R_1(C_n)$, we see that

$$R_1(G) = \sum_{uv\in E(G)} |r_G(u) + r_G(v)|$$
$$= \sum_{RE_{22}} |r_G(u) + r_G(v)|$$
$$= n(2+2)$$
$$R_1(G) = 4n$$

2. To compute $R_2(C_n)$, we see that

$$R_2(G) = \sum_{uv\in E(G)} |r_G(u) r_G(v)|$$
$$R_2(G) = \sum_{RE_{22}} |r_G(u) r_G(v)|$$
$$= n(2\times2)$$
$$R_2(G) = 4n$$

□

Theorem 3.4 Let C_n^2 is the square cycle graph then

1. $R_1(C_n^2) = 16n$
2. $R_2(C_n^2) = 32n$

Proof. Let G be the graph C_n^2. In the square cycle graph by the algebraic method there is one type of edges on the degree of end vertices as follows

$E_{44} = \{uv \in E(G) / d_G(u) = 4 \,\&\, d_G(v) = 4\},\ |E_{44}| = 2n$

Thus we have one type of revan edges based on the degree of the end revan vertices of each edge as follows, we have $\Delta(G)+\delta(G)=8$.

$RE_{44}=\{uv\in E(G)/r_G(u)=4\,\&\,r_G(v)=4\},\ |RE_{44}|=2n$

1. To compute $R_1(C_n^2)$, we see that

$$R_1(G)=\sum_{uv\in E(G)}|r_G(u)+r_G(v)|$$
$$=\sum_{RE_{44}}|r_G(u)+r_G(v)|$$
$$=2n(4+4)$$
$$R_1(G)=16n$$

2. To compute $R_2(C_n^2)$, we see that

$$R_2(G)=\sum_{uv\in E(G)}|r_G(u)r_G(v)|$$
$$R_2(G)=\sum_{RE_{44}}|r_G(u)r_G(v)|$$
$$=2n(4\times 4)$$
$$R_2(G)=32n$$

□

Theorem 3.5 Let $W_{1,n}$ is the wheel graph then

1. $R_1(W_{1,n})=3n(n+1)$
2. $R_2(W_{1,n})=n^2(n+3)$

Proof. Let G be the graph $W_{1,n}$. In the wheel graph by the algebraic method there are two types of edges on the degree of end vertices as follows

$E_{3n}=\{uv\in E(G)/d_G(u)=3\,\&\,d_G(v)=n\},\ |E_{3n}|=n$

$E_{33}=\{uv\in E(G)/d_G(u)=3\,\&\,d_G(v)=3\},\ |E_{33}|=n$

Thus we have two types of revan edges based on the degree of the end revan vertices of each edge as follows, we have $\Delta(G)+\delta(G)=n+3$.

$RE_{3n}=\{uv\in E(G)/r_G(u)=3\,\&\,r_G(v)=n\},\ |RE_{3n}|=n$

$RE_{nn}=\{uv\in E(G)/r_G(u)=n\,\&\,r_G(v)=n\},\ |RE_{nn}|=n$

1. To compute $R_1(W_{1,n})$, we see that

$$R_1(G)=\sum_{uv\in E(G)}|r_G(u)+r_G(v)|$$
$$=\sum_{RE_{3n}}|r_G(u)+r_G(v)|+\sum_{RE_{nn}}|r_G(u)+r_G(v)|$$
$$=n(3+n)+n(n+n)$$
$$=3n+n^2+2n^2$$
$$R_1(G)=3n(n+1)$$

2. To compute $R_2(W_{1,n})$, we see that

$$R_2(G)=\sum_{uv\in E(G)}\left|r_G(u)r_G(v)\right|$$

$$R_2(G)=\sum_{RE_{3n}}\left|r_G(u)r_G(v)\right|+\sum_{RE_{nn}}\left|r_G(u)r_G(v)\right|$$

$$=n(3n)+n(n\times n)$$

$$=3n^2+n^3$$

$$R_2(G)=n^2(n+3)$$

□

Theorem 3.5 Let F_n is the fan graph then

1. $R_1(F_n)=3n^2-3n+6$
2. $R_2(F_n)=n^3-n^2+3n+1$

Proof. Let G be the graph F_n. In the fan graph by the algebraic method there are four types of edges on the degree of end vertices as follows

$$E_{2n}=\{uv\in E(G)/d_G(u)=2\,\&\,d_G(v)=n\},\ |E_{2n}|=2$$

$$E_{3n}=\{uv\in E(G)/d_G(u)=3\,\&\,d_G(v)=n\},\ |E_{3n}|=n-2$$

$$E_{23}=\{uv\in E(G)/d_G(u)=2\,\&\,d_G(v)=3\},\ |E_{23}|=2$$

$$E_{33}=\{uv\in E(G)/d_G(u)=3\,\&\,d_G(v)=3\},\ |E_{33}|=n-3$$

Thus we have four types of revan edges based on the degree of the end revan vertices of each edge as follows, we have $\Delta(G)+\delta(G)=n+2$.

$$RE_{2n}=\{uv\in E(G)/r_G(u)=2\,\&\,r_G(v)=n\},\ |RE_{2n}|=2$$

$$RE_{2(n-1)}=\{uv\in E(G)/r_G(u)=2\,\&\,r_G(v)=n-1\},\ \left|RE_{2(n-1)}\right|=n-2$$

$$RE_{(n-1)(n-1)}=\{uv\in E(G)/r_G(u)=n-1\,\&\,r_G(v)=n-1\},\ \left|RE_{(n-1)(n-1)}\right|=n-3$$

$$RE_{n(n-1)}=\{uv\in E(G)/r_G(u)=n\,\&\,r_G(v)=n-1\},\ \left|RE_{n(n-1)}\right|=2$$

1. To compute $R_1(F_n)$, we see that

$$R_1(G)=\sum_{uv\in E(G)}\left|r_G(u)+r_G(v)\right|$$

$$=\sum_{RE_{2n}}\left|r_G(u)+r_G(v)\right|+\sum_{RE_{2(n-1)}}\left|r_G(u)+r_G(v)\right|+\sum_{RE_{(n-1)(n-1)}}\left|r_G(u)+r_G(v)\right|+\sum_{RE_{n(n-1)}}\left|r_G(u)+r_G(v)\right|$$

$$=2(2+n)+(n-2)(2+n-1)+(n-3)(n-1+n-1)+2(n+n-1)$$

$$=(2n+4)+(n^2-n-2)+(2n^2-8n+6)+(4n-2)$$

$$R_1(G)=3n^2-3n+6$$

2. To compute $R_2(F_n)$, we see that

$$R_2(G)=\sum_{uv\in E(G)}\left|r_G(u)r_G(v)\right|$$

$$R_2(G)=\sum_{RE_{2n}}\left|r_G(u)r_G(v)\right|+\sum_{RE_{2(n-1)}}\left|r_G(u)r_G(v)\right|+\sum_{RE_{(n-1)(n-1)}}\left|r_G(u)r_G(v)\right|+\sum_{RE_{n(n-1)}}\left|r_G(u)r_G(v)\right|$$

$$=2(2\times n)+(n-2)(2\times(n-1))+(n-3)(n-1)^2+2(n\times(n-1))$$

$$=4n+(n-2)(2n-2)+(n-3)(n^2-2n+1)+2n^2-2n$$

$$=4n+(2n^2-6n+4)+(n^3-5n^2+7n-3)+2n^2-2n$$

$$R_2(G)=n^3-n^2+3n+1$$

□

Theorem 3.6 Let C_n+K_1 is the corona graph then

1. $R_1(C_n+K_1)=6n$
2. $R_2(C_n+K_1)=4n$

Proof. Let G be the graph C_n+K_1. In the corona graph by the algebraic method there are two types of edges on the degree of end vertices as follows

$$E_{13}=\{uv\in E(G)/d_G(u)=1\,\&\,d_G(v)=3\},\ |E_{13}|=n$$

$$E_{33}=\{uv\in E(G)/d_G(u)=3\,\&\,d_G(v)=3\},\ |E_{33}|=n$$

Thus we have two types of revan edges based on the degree of the end revan vertices of each edge as follows, we have $\Delta(G)+\delta(G)=4$.

$$RE_{13}=\{uv\in E(G)/r_G(u)=1\,\&\,r_G(v)=3\},\ |RE_{13}|=n$$

$$RE_{11}=\{uv\in E(G)/r_G(u)=1\,\&\,r_G(v)=1\},\ |RE_{11}|=n$$

1. To compute $R_1(C_n+K_1)$, we see that

$$R_1(G)=\sum_{uv\in E(G)}\left|r_G(u)+r_G(v)\right|$$

$$=\sum_{RE_{31}}\left|r_G(u)+r_G(v)\right|+\sum_{RE_{11}}\left|r_G(u)+r_G(v)\right|$$

$$=n(3+1)+n(1+1)$$

$$R_1(G)=6n$$

2. To compute $R_2(C_n+K_1)$, we see that

$$R_2(G)=\sum_{uv\in E(G)}\left|r_G(u)r_G(v)\right|$$

$$R_2(G)=\sum_{RE_{31}}\left|r_G(u)r_G(v)\right|+\sum_{RE_{11}}\left|r_G(u)r_G(v)\right|$$
$$=n(3\times1)+n(1\times1)$$
$$=3n+n$$
$$R_2(G)=4n$$

□

Theorem 3.7 Let P_n+K_1 is the comb graph then

1. $R_1(P_n+K_1)=6n+2 \qquad n\geq 4$
2. $R_2(P_n+K_1)=4n+7 \qquad n\geq 4$

Proof. Let G be the graph P_n+K_1. In the comb graph by the algebraic method there are four types of edges on the degree of end vertices as follows

$$E_{12}=\{uv\in E(G)/d_G(u)=1\,\&\,d_G(v)=2\},\ |E_{12}|=2$$
$$E_{13}=\{uv\in E(G)/d_G(u)=1\,\&\,d_G(v)=3\},\ |E_{13}|=n-2$$
$$E_{23}=\{uv\in E(G)/d_G(u)=2\,\&\,d_G(v)=3\},\ |E_{23}|=2$$
$$E_{33}=\{uv\in E(G)/d_G(u)=3\,\&\,d_G(v)=3\},\ |E_{33}|=n-3$$

Thus we have four types of revan edges based on the degree of the end revan vertices of each edge as follows, we have $\Delta(G)+\delta(G)=4$.

$$RE_{32}=\{uv\in E(G)/r_G(u)=3\,\&\,r_G(v)=2\},\ |RE_{32}|=2$$
$$RE_{21}=\{uv\in E(G)/r_G(u)=2\,\&\,r_G(v)=1\},\ |RE_{21}|=2$$
$$RE_{31}=\{uv\in E(G)/r_G(u)=3\,\&\,r_G(v)=1\},\ |RE_{31}|=n-2$$
$$RE_{11}=\{uv\in E(G)/r_G(u)=1\,\&\,r_G(v)=1\},\ |RE_{11}|=n-3$$

1. To compute $R_1(P_n+K_1)$, we see that

$$R_1(G)=\sum_{uv\in E(G)}\left|r_G(u)+r_G(v)\right|$$
$$=\sum_{RE_{32}}\left|r_G(u)+r_G(v)\right|+\sum_{RE_{31}}\left|r_G(u)+r_G(v)\right|+\sum_{RE_{21}}\left|r_G(u)+r_G(v)\right|+\sum_{RE_{11}}\left|r_G(u)+r_G(v)\right|$$
$$=2(3+2)+(n-2)(3+1)+2(2+1)+(n-3)(1+1)$$
$$=10+(4n-8)+6+(2n-6)$$
$$R_1(G)=6n+2 \qquad n\geq 4$$

2. To compute $R_2(F_n)$, we see that

$$R_2(G)=\sum_{uv\in E(G)}\left|r_G(u)r_G(v)\right|$$

$$R_2(G) = \sum_{RE_{32}} |r_G(u) r_G(v)| + \sum_{RE_{31}} |r_G(u) r_G(v)| + \sum_{RE_{21}} |r_G(u) r_G(v)| + \sum_{RE_{11}} |r_G(u) r_G(v)|$$
$$= 2(3\times 2) + (n-2)(3\times 1) + 2(2\times 1) + (n-3)(1\times 1)$$
$$= 12 + (3n-6) + 4 + (n-3)$$
$$R_2(G) = 4n + 7 \qquad n \geq 4$$

□

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