Generalization of Vedic Mathematics Method for Nth Degree Polynomial Factorization: Using Sutras Paravartya Yojayet, Vilokanam and Ganita Samuccaya: Samuccaya Gunita

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Abstract: In this paper, we have presented a new approach to factorization of apolynomial,with the help of Vedic mathematics and modern mathematics concepts. This method is based on the Vedic mathematics sutras Vilokanam, Gunita Sumaccaya: Samuccaya Gunita, and Paravartya Yojayet given by Swami Bharti Krishna Maharaj. It is an easier and more useful method for studentsto easily factorize this type of polynomial with mental calculation.

Keywords: Modern Mathematics, Fundamental Theorem of Algebra, Vedic Mathematics, Vilokanam, Gunita Samuccaya: Samuccaya Gunita, Paravartya Yojayet, polynomial.

Introduction:

Vedic mathematics is an ancient Indian system of mathematical calculations or operation techniques that was developed in 1957. It consists of 16 sutras (formulae) and 13 sub-sutras (Sub Formulae) that can be utilized to solve problems in arithmetic, algebra, geometry, calculus, and conics. [1]. Various methods have been developed for the factorization of polynomials. In the Vedic method by Jagadguru Shankaracharya Shri Bharati Krishna Thirtha Swamiji, He introduced the method of factoring cubic polynomials [1]. In this study, we use the same concept for factorization of the nth type of polynomial equation with the help of modern mathematics.

The introduction of a method for factorizing nth-degree polynomials, which can be expressed as $x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_k,$ (where x is variable of polynomial, n is degree of polynomial and$a_1, a_2, \ldots, a_k$ are constant terms of polynomial)

Type (1): When thepolynomial doesnot have a factor $(x-1)$**and**$(x+1)$**.**

Step (1)
If the Sum of the coefficients of this polynomial$s_c = 0$.Then one factor of the polynomial is $(x-1)$ and if the sum of the coefficient of odd power of the polynomials_0 = sum of the coefficient of even power of the polynomial s_e. Then one factor is $(x+1)$. If both conditions are not satisfied. Then the given polynomial does not have $(x+1)$ and $(x-1)$ factors.

Step (2)
The polynomial equation is of the nth degree. So, By the Fundamental Theorem of Algebra, every polynomial equation of n degree has n roots. Here, the polynomial is nth degree, so it has nth roots called, them $a_1 a_2 a_3, \ldots, and\ a_n$.

Hence
$A = a_1 + a_2 + a_3 + \cdots + a_n$ = Sum of roots = coefficient of x^n …Condition (1)
$B = a_1 a_2 a_3, \ldots, a_n$ = Product of four roots = last term .... Condition (2)

Step (3)
Write down all possible factors of the last term of the given polynomial without 1. As in Step (1) we said that$(x+1)$ and$(x-1)$ are not factors of the given polynomial.
(Note that here we take the last term as only a positive number and write all the positive factors of the last term)

Step (4)
Apply Vilokanam Sutras (mere observation) for condition $B = a_1 a_2 a_3, \ldots, a_n$. (Assume B as a positive number)
Observe Step 3 and make possible groups of the nth term that satisfy the condition. And write down all the possible groups.

Step (5)
1) If only one group satisfies Step (4), then that group factor is the factor of the given polynomial.
2) If more than one group of factors satisfies Step (4), Then use (+ or – sing) with factor check condition number (1).
3) From those groups, if one group satisfies the condition (1), Then that group factor with their sign is a factor of the given polynomial.
4) If more than one group also satisfies the condition (1) with their sign. Then check those groups with their (+ or -) sign condition (2). When checking the condition, if only one group satisfies the condition. Then this group, with their sign all factors are factors of the given polynomial.

If more than one group satisfies condition (2) then try Step (6).

Step (6)
Here by Gunita Samucaaya: Samucaaya gunita, we can verify that the factors are right or wrong and check by these Steps which factors groups are factors of the given polynomial.
By checking the Sum of the coefficients in the product of the polynomial =Product of the sum of the coefficients of factors.

Example: Factories$x^3 - 8x^2 - 5x + 84$

Step (1)
The sum of the coefficient $s_c = 1 - 8 - 5 + 84 = 72$, $s_c \neq 0$.

So $(x-1)$ is not a factor of the given polynomial.
Also, the sum of the coefficient of odd power $s_0 = 1 - 5 = -4$.
The sum of thecoefficient of even power $s_0 = -8 + 84 = 76$.
The sum of the coefficient of odd power s_0 ≠sum of the coefficient of even power s_e.
Therefore $(x+1)$ is not a factor of the given polynomial.

Step (2)
The polynomial is three degrees. So, it has three roots called k_1, k_2 *and* k_3.

Hence
$A = k_1 + k_2 + k_3$ =Sum of roots = coefficient of $x^2 = -8$... (1)
$B = k_1k_2k_3$ = Product of four roots = 84.... (2)

Step (3)
For the given polynomial, the last term is 84 (we take the last term as positive). So, factories 84 without 1.

$$2, 3, 4, 6, 7, 12, 14 \text{ and } 21.$$

Step (4)
From these factors, there are possible groups that satisfy the condition $B = k_1k_2k_3$.
Group 1: $(2 \times 2 \times 21) = 84$
Group 2: $(2 \times 3 \times 14) = 84$
Group 3: $(3 \times 4 \times 7) = 84$
Group 4: $(2 \times 6 \times 7) = 84$
So here four groups satisfy this step.

Step (5)
Check condition (1)
Here only satisfies cases $(3, -4, -7)$Satisfies Step 2 conditions (1)

$$A = k_1 + k_2 + k_3 = 3 - 4 - 7 = -8$$

Check this case for Step 2 condition (2)

$$B = (3)(-4)(-7) = 84$$

Here in this case$(3, -4, -7)$Both conditions satisfy so the factors of the given polynomial are $(x+3, x-4, x-7)$

Step (6)
Here, The sum of the coefficient product of the polynomial= 72
Product of the sum of the coefficients of factor$(1+3)(1-4)(1-7) = 72$.
Here the sum of the coefficient of product= product of the sum of the coefficient of factors.
Hence result verified by Gunita Samuccaya: Samuccaya Gunita.
$\therefore x^3 - 8x^2 - 5x + 84 = (x+3), (x-4), (x-7)$.

Example: Factories$x^4 + 6x^3 - 25x^2 - 222x - 360$

Step (1)
Here, $s_c = -600 \neq 0$ and$s_0 = -216$, $s_e = -384$, and $s_0 \neq s_e$.
Therefore, $(x-1)$ and $(x+1)$ are no factors in the given polynomial.

Step (2)
This polynomial is of the fourth degree. So, it has four roots called them $p_1 p_2 p_3$ *and* p_4

Hence
$A = p_1 + p_2 + p_3 + p_4$ =Sum of roots = coefficient of $x^3 = 6$... (1)
$B = p_1p_2p_3p_4$ = Product of four roots $= -360$... (2)

Step (3)
For the given polynomial, the last term is 360 (we take the last term as positive). So, factories 360 without 1
I.e. (2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120,180, 360).

Step (4)
From these factors, possible groups that satisfy the condition $B = p_1p_2p_3p_4$

$(45 \times 2 \times 2 \times 2) = 360$, $(30 \times 3 \times 2 \times 4) = 360$, $(20 \times 3 \times 3 \times 2) = 360$

$(18 \times 5 \times 2 \times 2) = 360$, $(15 \times 6 \times 2 \times 2) = 360$,$(15 \times 4 \times 3 \times 2) = 360$

$(12 \times 5 \times 3 \times 2) = 360$, $(10 \times 9 \times 2 \times 2) = 360$,$(10 \times 6 \times 3 \times 2) = 360$

$(9 \times 5 \times 4 \times 2) = 360$, $(8 \times 5 \times 3 \times 3) = 360$,$(6 \times 6 \times 5 \times 2) = 360$

$(6 \times 5 \times 4 \times 3) = 360$.

Step (5)
Check condition (1)
Case (1)

$$A = p_1 + p_2 + p_3 + p_4 = 6$$
$$= 15 - 4 - 3 - 2 = 6$$

Therefore, case 1 four-factor group is $(15, -4, -3, -2)$
Case (2)

$$A = p_1 + p_2 + p_3 + p_4 = 6$$
$$= 12 - 5 - 3 + 2 = 6$$

Therefore, case 2 four-factor group is $(12, -5, -3, 2)$
Case (3)

$$A = p_1 + p_2 + p_3 + p_4 = 6$$
$$= 9 - 5 + 4 - 2 = 6$$

So, case 3 the factor group is $(9, -5, 4, -2)$
Case (4)

$$A = p_1 + p_2 + p_3 + p_4 = 6$$
$$= -6 + 5 + 4 + 3 = 6$$

Therefore, case 4 the factor group is $(-6, 5, 4, 3)$

In these four cases, all four factors satisfy these conditions
Therefore, we check all four groups from which case satisfies condition (2)

Here cases (1) and case (4) satisfy condition (2)
Case (1)

$$B = (15 \times -4 \times -3 \times -2) = -360$$

Case (4)

$$B = (-6 \times +5 + 4 \times +3) = -360$$

Therefore, four factors of a given polynomial are $(x + 15), (x - 4), (x - 3), (x - 2)$

otherwise, four factors of the given polynomialare$(x-6),(x+5),(x+4),(x+3)$.

Step (6)
So here, by using sutra Gunita Samuccaya:Samuccaya Gunita.
We can check the Sum of the coefficients in the product of polynomials =Product of the sum of the coefficients of factors.

The sum of the coefficient product of the given polynomial is $= -600$

Here first we Check the condition for factors$(x+15),(x-4),(x-3),(x-2)$
Product of the sum of the coefficients of factors.

$$= (1+15)(1-4)(1-3)(1-2) = (16)(-3)(-2)(-1) = -96$$

So here factor groups $(x+15),(x-4),(x-3),(x-2)$is is not a factor of the given polynomial

For factors, $(x-6),(x+5),(x+4),(x+3)$
The Product of the sum of the coefficients of factors

$$= (1-6)(1+5)(1+4)(1+3) = (-5)(6)(5)(4) = -600$$

Here, this all-factor group satisfies Step 6 conditions.
The sum of the coefficients of the product = product of the sum of the coefficients of the factors.

$$\therefore x^4 + 6x^3 - 25x^2 - 222x - 360 = (x-6)(x+5)(x+4)(x+3)$$

Type (2): When the polynomial has $(x-1)$either$(x+1)$factors,otherwise if both $(x-1)$and $(x+1)$are factors of the given polynomial.

1) If the polynomial satisfies the Step 1 condition in the Type (1) Method.Then the polynomial is divided by$(x-1)$If $s_c = 0$, otherwise if $s_0 = s_e$Then divided by$(x+1)$.Otherwise if both Step 1 conditions are satisfied then the polynomial is divided by both $(x-1)$ *and* $(x+1)$Using paravartyayojayet. After dividing among factors by polynomial then the remaining Quotient is the new polynomial.

2) By using the new polynomial, we can find the Remaining factor of the polynomial by using the above Type-1 method.

Example: Factories $x^5 + 3 \cdot x^4 - 113x^3 - 447 \cdot x^2 - 104 \cdot x + 660$

Step (1)
Here $s_c = 0$ and$s_0 \neq s_e$.
Therefore, $(x-1)$ is one factor of the given polynomial.

By using Paravartya sutras, divide the polynomial by $(x-1)$

$x-1$ $MD\ 1$	$1 \quad +3 \quad -113 \quad -447 \quad -104 \quad : +660$ 1 4 -109 -556 $: -556$
	$1 \quad +4 \quad -109 \quad -556 \quad -660 \quad : \quad 0$

New Polynomial (Quotient)= $x^4 + 4x^3 - 109x^2 - 556x - 660$

(Note: here we will use Type (1) method on the new polynomial to get the remaining factors of the given polynomial)

Step (1)
Here $s_c \neq 0$ and$s_0 \neq s_e$.
Therefore $(x-1)$ and $(x+1)$ are not factored into the given new polynomial.

Step (2)
The new polynomial is of the fourth degree. So, it has four roots called them $n_1, n_2, n_3 and\ n_4$

Hence
$A = n_1 + n_2 + n_3 + n_4 = 4 \ldots (2)$
$B = n_1 n_2 n_3 n_4 = -660 \ldots (3)$

Step (3)
Here are factors of 660 (without 1)

$$2, 3, 4, 5, 6, 10, 11, 12, 15, 20, 22, 30, 33, 44, 55, 60, 66, 110, 132, 165, 220, 330\ and\ 660.$$

Step (4)
From these factors, there are possible groups that satisfy the condition. $B = n_1 n_2 n_3 n_4$

$(55 \times 3 \times 2 \times 2) = 660$, $(33 \times 5 \times 2 \times 2) = 660$, $(22 \times 5 \times 3 \times 2) = 660$
$(15 \times 11 \times 2 \times 2) = 660$, $(11 \times 10 \times 3 \times 2) = 660$, $(11 \times 5 \times 6 \times 2) = 660$
$(11 \times 5 \times 4 \times 3) = 660$,
So here, seven groups satisfy this condition.

Step (5)
Check condition (1)

Case (1)

$$A = 15 - 11 + 2 - 2 = 4$$

Therefore, case 1 four-factor group is $(15, -11, 2, -2)$

Case (2)

$$A = -11 + +10 + 3 + 2 = 4$$

Therefore, case 2 four-factor group is $(-11, 10, 3, 2)$

In these two cases, all four factors satisfy these conditions.

Therefore, we check all four groups to see which case satisfies condition (2).

Here only case (2) satisfies condition (2)

Case (2)

$$B = (-11 \times 10 \times 3 \times 2) = 660$$

Therefore, factors of the given new polynomial are $(x - 11,\ x + 10,\ x + 3,\ x + 2)$

So, all factors of the given main polynomial

$x^5 + 3 \cdot x^4 - 113x^3 - 447 \cdot x^2 - 104 \cdot x + 660$is $(x - 11, x + 10, x + 3,\ x + 2, x - 1)$

Step (6)

Here, byGunita Samuccaya:Samuccaya Gunita verify.

The sum of the coefficient product of the main polynomial = 0

Product of the sum of the coefficients of factors

$$(x - 11)(x + 10)(x + 3)(x + 2)(x - 1) = 0.$$

The sum of the coefficients of the product of the sum of the coefficient of factors.

Hence, result verified by Gunita Samuccaya: Samuccaya Gunita.

Conclusion:

On the basis of the above discussion, we may conclude that the Vedic mathematics method of factorization of polynomials is a mental calculation technique that requires few algebraic procedures, making it faster than the standard method. In this study, we apply the method to cubic, four-degree, and five-degree polynomials. Using these methods, we can also find the nth degree polynomial equation. This method can factorize only a particular polynomial,which can be expressed as $x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_k$.

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