Scope
Volume 13 Number 4 December 2023

FRW Minimally Interacting Holographic Dark Energy Cosmological Model in a Scalar – Tensor Theory of Gravitation

Muddada Ramanamurty [1], R. Santhi kumar[2] , Sobhan Babu Kappala[3],

[1 & 2]Aditya Institute of Technology and Management, Tekkali, Srikakulam Dist Andhra Pradesh-India
[3]University Collage of Engineering, Narasaraopeta (Affiliated to JNTUK Kakinada), Andhra Pradesh-India,
[1]ORCID: 0009-0009-4086-486X, [2]ORCID: 0000-0001-5122-3800 [3]ORCID: 0000-0002-2991-7651

Abstract: This paper explores the evolutionary dynamics of the dark energy parameter within the spatially homogeneous and isotropic Friedmann-Robertson-Walker (FRW) model, incorporating both barotropic fluid and dark energy. The investigation is conducted within the framework of the scalar-tensor theory of gravitation proposed by Saez and Ballester (Phys. Lett. A 113:467, 1986). A well-defined solution is obtained by employing the special law of variation for Hubble's parameter, as put forth by Bermann (Nuovo Cimento B 74:183, 1983). Minimally Interacting Holographic Dark Energy Cosmological Model involving barotropic and dark energy, are considered, resulting in general outcomes. The paper also delves into the physical implications of the obtained results.

Key Wards: Holographic Dark energy, Scalar-tensor theory, , Minimally Interacting.

1. Introduction

In recent years there has been an immense interest in cosmological models with dark energy in general relativity because of the fact that our observable universe is undergoing a phase of accelerated expansion which has been confirmed by several cosmological observations such as type 1a supernova [1-7]. Cosmic microwave background (CMB)anisotropy [8,9]and large scale structure [10] strongly indicate that dark energy dominates the present universe, causing cosmic acceleration. Based on these observations, cosmologists have accepted the idea of dark energy, which is a fluid with negative pressure making up around 70% of the present universe energy content to be responsible for this acceleration due to repulsive gravitation. Cosmologists have proposed many candidates for dark energy to fit the current observations such as cosmological constant, tachyon, quintessence, phantom and so on. For instance, quintessence models involving scalar fields give rise to time dependent equation of state(EoS) parameter $\omega = p/\rho$ which is not necessarily constant where p is the fluid pressure and ρ is energy density[11] . Some of the authors [12-21]who have investigated several aspects of dark energy models in general relativity with variable EoS parameter.

It is well know that there are two major approaches to address the problem of late time acceleration of the universe. One approach is by introducing a dark energy component in the universe and study its dynamics. An alternative approach is modifying the general relativity itself [22-25]. This is known as 'modified gravity approach'. In spite of the fact that both approaches have noble features with some deep theoretical problems we , here , focus our attention on the modified gravity approach. Brans- Dicke gravity[26], which introduces, in addition to the metric tensor field, a dynamical scalar field to account for variable gravitational constant, was one of the earlier modification of general relativity. This modification was introduced due to lack of compatibility of Einstein's theory with the mach's Principle. Later Saez and Ballester[27]have formulated a scalar – tensor theory of gravity in which metric is coupled to a scalar field.

This modification helped to solve the “missing mass problem”. Several aspects of Saez – Ballester theory in relation to Bianchi Cosmological models have been explored[28-31]. In particular, Bianchi type dark energy cosmological models have been investigated by several authors [32-35].

This Among the many different approaches to describe the dark energy cosmological models, holographic dark energy models have received considerable attention. A new alternative to the solution of dark energy problem may be found using ‘Holographic principle. The holographic dark energy model is an emerging model as a candidate of dark energy constructed by holographic principle [36-40]. It is understood that these models may solve the cosmological constant problem and some other issues. Some cosmologists have studied several aspects of holographic dark energy. There are many reports on the investigation of several aspects of holographic dark energy. [41-44] have studied holographic dark energy model in BD theory. Setare and Vagenas [45] have discussed cosmological dynamics of interacting holographic dark energy model. Das and Mammon have [46] discussed an interacting model of dark energy in BD theory. Recently Sarkar and Mahanta [47] have obtained holographic dark energy model in Bianchi type-I spacetime. Sarkar [48-49] have investigated interacting holographic dark energy model in Bianchi type-V universe, while Adhav et al. [50] have discussed holographic interacting dark energy anisotropic models. Very recently, Reddy et al.[51] obtained Bianchi type-V minimally interacting holographic dark energy model in the scalar-tensor theory of gravitation proposed by Saez and Ballester. Samantha [52] has obtained holographic dark energy cosmological model with quintessence in Bianchi type-V space-time. Kiran et al. [53] have investigated Bianchi type-V minimally interacting holographic dark energy models in Brans-Dicke and Saez Ballester theories of gravitation. Recently, Reddy et al. [54] have obtained Kaluza-Klein minimally interacting holographic dark energy model in a scalar-tensor theory of gravitation of the universe. Motivated by the above investigation and discussions, we have investigated FRW Minimally Interacting Holographic Dark Energy Cosmological Model in Saez-Ballester Theory of Gravitation.

This chapter is structured as follows: Section .2 focuses on deriving the Saez-Ballester field equations, employing the spatially homogeneous and isotropic FRW metric. This derivation is carried out within the framework of the scalar-tensor theory of gravitation formulated by Saez and Ballester (1986), considering the presence of both barotropic fluid and dark energy. In Section .3 , d elves into the dynamics of minimal interacting two mattters. The derivation of corresponding cosmological models within this scalar-tensor theory is facilitated by the utilization of the special law of variation for Hubble's parameter, as proposed by Bermann (1983). Throughout these sections, we provide a comprehensive discussion on the behavior of physical and kinematical parameters within the specified models.

2 .Metric and Field Equations:

Assuming the universe to be homogeneous and isotropic, the FRW metric can be written as

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \quad (1)$$

where a(t) is the scale factor and k = -1, 0,+1 respectively for open, flat and closed models of the universe. The field equations given by Saez and Ballester (1986) for the combined scalar and tensor fields (with $8\pi G = 1$ and c = 1) are

$$R_{ij} - \frac{1}{2}g_{ij}R - w\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -(T_{ij} + \overline{T_{ij}}) \quad (2)$$

In equation (2) T_{ij} represents stress energy tensor of dark matter with density ρ_m and $\overline{T_{ij}}$ represents stress energy tensor of holographic dark matter with density ρ_λ , which are defined as

$$T_{ij} = \rho_m u_i u_j \tag{3}$$

and $\overline{T_{ij}} = (\rho_\lambda + p_\lambda)u_i u_j + g_{ij}\, p_\lambda$ (4)

and the scalar field satisfies the equation

$$2\phi^n \phi_{;i}^{,i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{5}$$

Also , we have

$$T_{;j}^{ij} = 0 \tag{6}$$

which is a consequence of the field equation (1) and (2). Here w and n are constants, T_{ij} is the two fluid energy momentum tensor consisting of dark energy and barotropic fluid and comma and semicolon denote partial and covariant differentiation respectively.

In a co-moving coordinate system Saez-Ballester field equations (2) – (6) for the metric (1), in the two fluid scenario, lead to

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} - \frac{w}{2}\phi^n\dot{\phi}^2 = -p_\lambda \tag{7}$$

$$3\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) + \frac{w}{2}\phi^n\dot{\phi}^2 = \rho_m + \rho_\lambda \tag{8}$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{n}{2}\frac{\dot{\phi}^2}{\phi} = 0 \tag{9}$$

$$\dot{\rho_m} + 3\frac{\dot{a}}{a}\rho_m + \dot{\rho_\lambda} + 3\frac{\dot{a}}{a}(\rho_\lambda + p_\lambda) = 0 \tag{10}$$

3. **Holographic Minimally Interacting Model**

The both components conserve separately , since we consider minimally interacting fields, so that we have

$$\dot{\rho_m} + 3\frac{\dot{a}}{a}\rho_m = 0 \tag{11}$$

$$\dot{\rho_\lambda} + 3\frac{\dot{a}}{a}(\rho_\lambda + p_\lambda) = 0 \tag{12}$$

Also, The equation of state (EoS) parameters of the dark matter and holographic dark matter are given by

$$\omega_m = \frac{p_m}{\rho_m} \quad \text{and} \quad \omega_\lambda = \frac{p_\lambda}{\rho_\lambda} \tag{13}$$

$$\dot{\rho_m} + 3\frac{\dot{a}}{a}\rho_m = 0 \tag{14}$$

$$\dot{\rho_\lambda} + 3\frac{\dot{a}}{a}(1 + \omega_\lambda)\rho_\lambda = 0 \tag{15}$$

Now, we solving the Saez – Ballester field equations in both the cases we determine $a(t)$, ρ_m , p_m , ρ_λ , p_λ, ω_m , $\omega_\lambda and \phi$ and then study their physical behavior.

We can observe that there is a structural difference between equations (14) and (15). In view of the fact that EoS parameter ω_m is constant ,while ω_λ is allowed to be function of time, integration of equation (15) leads to

$$\rho_m = \rho_0 a^{-3(1+\omega_m)} \tag{16}$$

Using equation (16) in the equations (7) and (8), we first obtain $p_\lambda and \rho_\lambda$, in terms of scale factor a(t) , as

$$\rho_\lambda = 3\frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2} + \frac{w}{2}\phi^n\dot{\phi}^2 - \rho_0 a^{-3(1+\omega_m)} \tag{17}$$

and

$$p_\lambda = -\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) + \frac{w}{2}\phi^n\dot{\phi}^2 - \rho_0\omega_m a^{-3(1+\omega_m)} \tag{18}$$

To ascertain the scale factor a(t)we adopt the distinctive law governing the variation of Hubble's parameter, as proposed by Bermann (1983). This law results in models of the universe characterized by a constant deceleration parameter. The constant deceleration parameter q is defined by:

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = constant \tag{19}$$

In cases where q > 0, the model experiences standard deceleration, while q < 0 signifies inflation or accelerated expansion of the universe. In our context, where we are specifically addressing the accelerated expansion of the universe, we consider q < 0. Subsequently, the integration of equation (19) results in the solution

$$a(t) = (ct + d)^{1/1+q} \tag{20}$$

where c ≠ 0 and d are constants of integration and 1 + q > 0 for accelerated expansion of the universe, i.e. -1 < q < 0.

By a suitable choice of constants (we choose d=0,c=1) , we can write the metric (1), with the help of (20), as

$$ds^2 = -dt^2 + t^{\frac{2}{1+q}}\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \tag{21}$$

The model given by equation (21) represents non – interacting two fluid model in Saez – Ballester theory with the following physical properties . Integrating equation (9) and using equation (21), the scalar field in the model is given by

$$\phi^{\frac{n+2}{2}} = \phi_0\left(\frac{n+2}{2}\right)\left(\frac{1+q}{q-2}\right)t^{\frac{q-2}{1+q}} \tag{22}$$

Using equations (21) and (22) in equations (17) and (18) we obtain $p_\lambda and \rho_\lambda$ as

$$\rho_\lambda = \frac{3}{(1+q)^2}\frac{1}{t^2} + \frac{3k}{t^{2/1+q}} + \frac{w}{2}\frac{\phi_0^2}{t^{6/1+q}} - \frac{\rho_0}{t^{\frac{3(1+\omega_m)}{1+q}}} \tag{23}$$

$$p_\lambda = -\left[\frac{2}{1+q}\frac{1}{t^2} + \frac{3}{(1+q)^2}\frac{1}{t^2} - \frac{k}{t^{2/1+q}} + \frac{w}{2}\frac{\phi_0^2}{t^{6/1+q}} + \frac{\rho_0}{t^{\frac{3(1+\omega_m)}{1+q}}}\omega_m\right] \tag{24}$$

Using equations (23) and (24) in equation (12) we obtain

$$\omega_\lambda = -\left[\frac{\frac{1-2q}{(1+q)^2 t^2} - \frac{k}{2t^{2/1+q}} + \frac{w\phi_0^2}{2t^{6/1+q}} - \frac{\rho_0}{t^{\frac{3(1+\omega_m)}{1+q}}}\omega_m}{\frac{3}{(1+q)^2}\frac{1}{t^2} + \frac{3k}{t^{2/1+q}} + \frac{w}{2}\frac{\phi_0^2}{t^{6/1+q}} - \frac{\rho_0}{t^{\frac{3(1+\omega_m)}{1+q}}}}\right] \tag{25}$$

which is the equation of state (EoS) parameter of the halographic dark energy in terms of the cosmic time t.

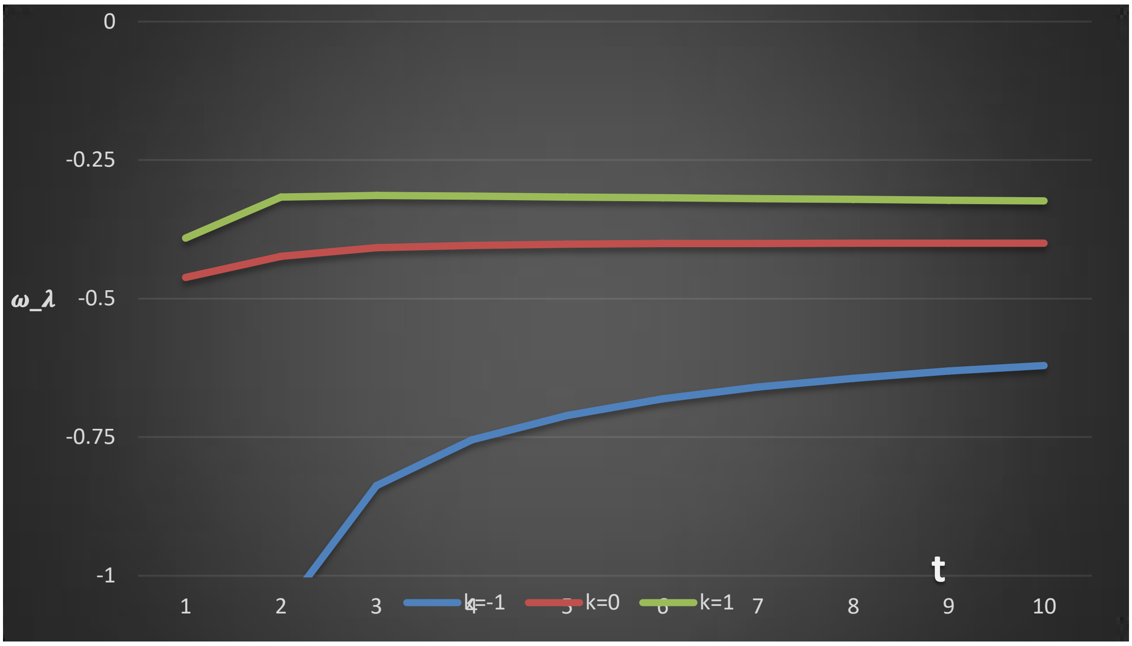

Fig 1.The plot of EoS Parameter ω_λ V_s . Cosmic time t. Here φ_0=1 , w=1, q= -0.1. ρ_0=1,ω_m=0.5,

As t approaches 0, it is evident that both , ρ_λ and p_λ diverge but as time progresses (for large t), they diminish. Equation (23) delineates the evolution of the equation of state (EoS) concerning cosmic time, t. The behavior of ω_λ in terms of cosmic time t is shown in figure-1, as depicted in Figure-1, indicates a consistent increase over time. The early-stage rate of growth depends on the universe type, with subsequent stabilization to a constant value.

Figure-1 reveals that the EoS parameters for closed, open, and flat universes exhibit variations in the quintessence region quintessence ($\omega_\lambda > -0.5$), phantom ($-1 < \omega_\lambda < -0.5$), and super phantom($\omega_\lambda > -0.3$)regions respectively

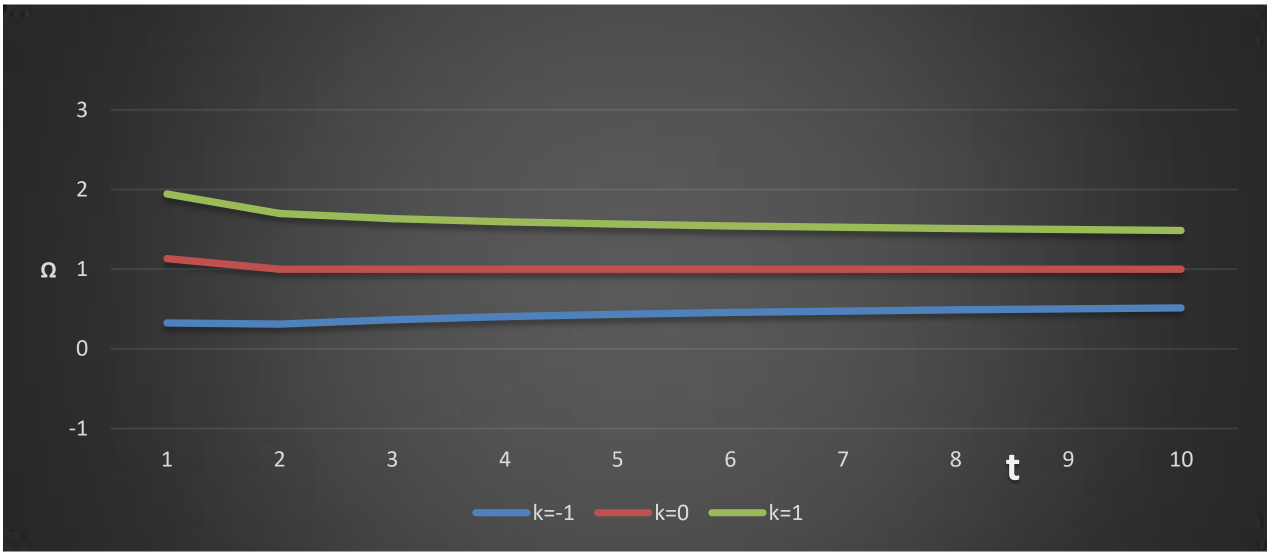

Fig2.The plot of average density parameter Ω V$_s$. Cosmic time t.Here ρ$_0$=1,ω$_m$=0.5, φ$_0$=1 , w=1, q= -0.1.

The expressions of the matter-density Ω_m and halographic dark-energy density Ω_λ are given by

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{\rho_0}{3}(1+q)^2\, t^{2\left(\frac{3(1+\omega_m)}{(1+q)}\right)} \tag{26}$$

$$\Omega_\lambda = \frac{\rho_\lambda}{3H^2} = 1 + (1+q)^2\left[kt^{\left(\frac{2q}{1+q}\right)} + \frac{w\phi_0^2}{6} t^{2\left(\frac{q-2}{(1+q)}\right)} - \frac{\rho_0}{3} t^{2\left(\frac{3(1+\omega_m)}{(1+q)}\right)}\right] \tag{27}$$

respectively.

Eq.(25) and (26) gives us the density parameter

$$\Omega = \Omega_m + \Omega_\lambda = 1 + (1+q)^2\left[kt^{\left(\frac{2q}{1+q}\right)} + \frac{w\phi_0^2}{6} t^{2\left(\frac{q-2}{(1+q)}\right)}\right] \tag{28}$$

We note that in a flat universe (k=0), Ω approaches 1, while in an open universe (k=-1), 0<Ω<1, and in a closed universe (k=1), Ω exceeds 1. However, at later times, we observe that, irrespective of the curvature (flat, open, or closed), Ω tends to approach 0. This observation aligns well with empirical findings. Given that our model predicts a flat universe for extended periods, and the present-day universe closely resembles a flat universe, our derived model concurs with observational results. The graphical representation of the density parameter's variation with cosmic time is illustrated in Figure 2.

4. **Conclusions**

In this investigation, we have FRW Minimally Interacting Holographic Dark Energy Cosmological Modelin the presence of the Saez–Ballester scalar field within the spatially homogeneous and isotropic FRW spacetime. Our findings reveal that the equation of state (EoS) parameter exhibits an upward trend with cosmic time across open, closed, and flat FRW universes, providing an explanation for the late-time acceleration of the universe. The exploration of scalar field dynamics in the context of an inflationary (accelerated) universe scenario holds significance as it offers potential solutions to lingering issues within standard 'big bang cosmology.' Notably, our analysis indicates that the current study, when applied to both open and flat universes, can traverse the phantom region. Additionally, the closed universe aligns with quintessence, while flat and open universes correspond to the phantom model of the universe. Throughout the evolution of the universe, the EoS parameters for the closed universe undergo a transition from ω > -1 to ω < -1, in accordance with recent observational findings. Our solutions are demonstrated to be both physically viable and stable

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