Two- and Three-Dimensional Partial Differential Equations Solved by using the DTM

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Abstract

In this work, using the differential transform approach, we get an approximate series solution for various partial differential equations in this study. By solving higher-order two- and three-dimensional partial differential equations, the DTM minimizes the amount of calculus effort.

Keywords: Differential transformed method, higher-order two- and three-dimensional.

Introduction:

The applying of DTM is effective for solving two-dimensional and three-dimensional PDEs with initial value problems. It causes scientific findings in the for mentioned study fields. It is critical to investigate different techniques for integrating these PDEs. [1-5] The DTM is a highly successful and efficient instrument for addressing both one-dimensional and multi-dimensional problems. Zhou established the notion of DTM initially in 1986 [1]. This method employs a sequential approach to generate analytical solutions in the form of polynomials based on the Taylor series expansion. Angalgil & Ayazat.al. in [2], used to solved the either liner along with non-linear differential equations such as the KdV and MKdV equations. Author in [2-4], presented, the two-point boundary value problem, while in authors in [5-9] presented the linear parabolic-hyperbolic PDE, and the two-dimensional nonlinear Gas dynamic, and the Klien-Gordon equations respectively.

The DTM approach is also used to resolve the three-dimensional linear Helmholtz problem in a specific form of results:

$$l\frac{\partial z^2}{\partial d^2} + m\frac{\partial z^2}{\partial e^2} + n\frac{\partial z^2}{\partial f^2} + \lambda z = A(d,e,f)$$

With the initial conditions:

$$z(0,e,f) = a_1(e,f) = z_d(0,e,f) = a_2(e,f)$$
$$z(d,0,f) = a_3(d,f) = z_e(d,0,f) = a_4(d,f)$$
$$z(d,e,0) = a_5(d,e) = z_f(d,e,0) = a_6(d,e)$$

Where $a_1(e,f), a_2(e,f), a_3(d,f), a_4(d,f), a_5(d,e), a_6(d,e)$ and l, m, n, λ are given function and constant respective.

"These equation has wide applications in various filed such as electrical and mechanical engineering and physics. The solutions to these problems are recommended to the reader (Zwilinger, 1992 Burdenand and Faires, 1993) [4]. Jafari and Zabini solved the above equations by homotopy perturbation method and homotopy analysis method respectively. [6-8] (Jafari.et.al.2010b and 2010c). In this paper we apply DTM for Helmholtz equation and Schrodinger equations".

Basic Definitions of Multi-dimensional DTM:

We define - dimensional differential transform and fundamental operation of $z(x_1, x_2, \ldots x_m)$ as

$$Z(w_1, w_2 \ldots, w_m) = \frac{1}{w_1!\, w_2!, \ldots w_m!}\left[\frac{\delta^{w_1+w_2+\cdots,+w_m} Z(x_1, x_2 \ldots, x_m)}{\delta x_1^{w_1}, \delta x_2^{w_2}, \ldots., \delta x_m^{w_m},}\right]_{(0,0,\ldots,0)} \quad (0.1)$$

Where $Z(x_1, x_2 \ldots, x_m)$is original function and $Z(w_1, w_2 \ldots, w_m)$is transformed function. The differential inverse transform of $Z(x_1, x_2 \ldots, x_m)$ is defined as follows:

$$Z(x_1, x_2 \ldots, x_m) = \sum_{w_1-0}^{\infty} \sum_{w_2-0}^{\infty} \ldots \sum_{w_m-0}^{\infty} Z(w_1, w_2 \ldots, w_m) x_1^{w_1}, x_2^{w_2}, \ldots, x_m^{w_m} \quad (0.2)$$

And from eq. (0.1) and (0.2) we can assume

Theorem 1: If $z(d_1, d_2 \ldots, d_m) = \lambda f\,(d_1, d_2 \ldots, d_m)$ then

$$Z(w_1, w_2 \ldots, w_m) = \lambda f\,(w_1, w_2 \ldots, w_m)$$

Where, λ is constant.

Theorem 2: If

$$z(d_1, d_2 \ldots, d_m) = \frac{\partial f(d_1, d_2 \ldots, d_m)}{\partial p_1} \text{ then } Z(w_1, w_2 \ldots, w_m) = (w_i + 1) F\,(w_1, w_2, \ldots, (w_i + 1), \ldots, w_m)$$

Theorem 3: If

$$z(d_1, d_2 \ldots, d_m) = d_1^{h_1}, d_2^{h_2} \ldots d_m^{h_m} \text{ then } Z(w_1, w_2 \ldots, w_m) = \delta(w_1 - h_1)\delta(w_2 - h_2) \ldots \delta(w_m - h_m)$$

Where,

$$\delta(w_i - h_i) = \begin{cases} 1 & w_i = h_i \\ 0 & \text{otherwise} \end{cases}$$

Theorem 4: If

$$z(d_1, d_2 \ldots, d_m) = d_1^{h_1}, d_2^{h_2} \ldots \sin(a_{x_i} + b) \ldots d_m^{h_m} \text{ then}$$

$$Z(w_1, w_2 \ldots, w_m) = \delta(w_1 - w_2) \ldots \frac{a^{k_i}}{w_{i!}} \sin\left(\frac{w_i \pi}{2} + b\right) \ldots \delta(w_m - h_m)$$

Theorem 5: If

$$z(d_1, d_2 \ldots, d_m) = {d_1}^{h_1}, {d_2}^{h_2} \ldots \cos(a_{x_i} + b) \ldots {d_m}^{h_m} \text{ then } Z(w_1, w_2 \ldots, w_m)$$
$$= \delta(w_1 - k_2) \ldots \frac{a^{k_i}}{w_{i!}} \cos\left(\frac{w_i \pi}{2} + b\right) \ldots \delta(w_m - h_m)$$

Solving two numerical using DTM:

Ex. 1] Solve the two-dimensional Schrodinger equations:

$\frac{\partial^2 z}{\partial d^2} + \frac{\partial^2 z}{\partial e^2} - 4z = (24d^2 - 5d^4)\, cos(e)$(1)

With the initial condition z (0, e) =0, z_d(0, e) =0 (2)

The exact solution can be expressed as z (d, t) =$d^4 cos(e)$

Taking the differential transform of (1)

$$(w_1 + 2)(w_1 + 1)Z(w_1 + 2, w_2) + (w_2 + 2)(w_2 + 1)Z(w_1, w_2 + 2) - 4Z(w_1, w_2, w_3)$$
$$= 24\delta(w_1 - 2)\frac{1}{w_2!} cos\left(\frac{w_2 \pi}{2}\right) - 5\delta(w_1 - 4)\frac{1}{w_2!} cos\left(\frac{w_2 \pi}{2}\right) \quad (3)$$

From the initial condition given by eq. (2)

Z (0,w_2)=0

Z (1,w_2) =0, $w_2 = 0,1,2, \ldots$(4)

Put eq. (3) into eq. (4)

The systematic technique yields the following results

$$Z(w_1, w_2) = \begin{cases} \frac{1}{w_2!} cos\left(\frac{w_2 \pi}{2}\right) & , if\ w_1 = 4 \\ 0 & , o.w. \end{cases}$$

We obtained the series solution as

Z (d, e) = $\sum_{w_1=0}^{\infty} \sum_{w_2=0}^{\infty} Z(w_1, w_2) d^{w_1} e^{w_2} = d^4 cos(e)$

Which is the exact answer.

Ex. 2] Solve three-dimensional Helmholtz equations:

$\frac{\partial^2 z}{\partial d^2} + \frac{\partial^2 z}{\partial e^2} - \frac{\partial^2 z}{\partial f^2} - 8z = (24d^2 - 8e^4)\, cos(e) sin(f)$ (5)

With the initial condition z (0, e, r) =0, z_p (0, e, f) =0 (6)

The exact solution can be expressed as z (d, t) =$d^4 cos(e) sin(f)$

Taking the differential transform of (5)

$$(w_1 + 2)(w_1 + 1)Z(w_1 + 2, w_2, w_3) + (w_2 + 2)(w_2 + 1)Z(w_1, w_2 + 2, w_3)$$
$$- (w_3 + 2)(w_3 + 1)Z(w_1, w_2, w_3 + 2) - 8Z(w_1, w_2, w_3)$$
$$= 24\delta(w_1 - 2)\frac{1}{w_2!} cos\left(\frac{w_2 \pi}{2}\right)\frac{1}{w_3!} sin\left(\frac{w_3 \pi}{2}\right)$$
$$- 8\delta(w_1 - 4)\frac{1}{w_2!} cos\left(\frac{w_2 \pi}{2}\right)\frac{1}{w_3!} sin\left(\frac{w_3 \pi}{2}\right) \quad (7)$$

From the initial condition given by eq. (6)

Z (0,$w_{2,}\ w_3$)=0

Z (1,$w_{2,} w_3$) =0, $w_{2,} w_3 = 0,1,2, \ldots$(8)

Put eq. (7) into eq. (8)

The systematic technique yields the following results

Z (w_1, w_2, w_3) =0 , if $w_1 \neq 8$ & $w_{2,}$ w_3=0, 1, 2…

Z (8,$w_{2,}$ w_3) = $\frac{1}{w_2!}\cos(\frac{w_2\pi}{2})\frac{1}{w_3!}\sin(\frac{w_3\pi}{2})Ifw_{2,}$ w_3=0, 1, 2…

We obtained the series solution as

Z (d, e, f) = $\sum_{w_1=0}^{\infty}\sum_{w_2=0}^{\infty}\sum_{w_3=0}^{\infty} Z(w_1, w_2, w_3)\, d^{w_1} e^{w_2} f^{w_3} = d^4 \cos(e)\sin(f)$

Which is the exact answer.

Conclusion:

Partial differential equations in two-dimensional and three-dimensional both linear and nonlinear, have been solved effectively using the DTM. The example demonstrates that the current technique's results are great agreement with the exact answer. It appears that DTM is very effective and potent approach to locating mathematical solution to a large range of linear and nonlinear problems.

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