Scope
Volume 13 Number 3 September 2023

Relevance of Vedic Mathematics Ekanyunena Purvena and Ekadhikena Purvena Sutras in Calculus

Rohit Ranjan Lal[1], Dharmendra Kumar Yadav[2]
1. University Department of Mathematics, Lalit Narayan Mithila University, Darbhanga, Bihar, India
2. University Department of Mathematics, Lalit Narayan Mithila University, Darbhanga, Bihar, India
Corresponding Author Email Id.: mdrdkyadav@gmail.com

Abstract
Problem: It is believed that Vedic Mathematics sutras are applicable for particular cases only in computation, whereas this study shows that the sutras have not been explored properly and if it is explored, a miracle may come out. **Approach:** In the present article we have analyzed the Vedic Mathematics Ekanyunena Purvena and Ekadhikena Purvena Sutras in finding the derivatives and antiderivatives of some elementary functions expressed in a power series using the power rule. We have also discussed the limitations of these sutras in finding derivative and antiderivative. For verification purpose, we have used Mathematica software to find the series of some functions, which can be applied for all elementary functions. **Findings:** Although it will be more laborious and time taking because to apply Vedic sutras, we have to follow four steps: to find series expansion, do term-wise operations, modify the new series to adjust it in some elementary functions and then write the result. In all examples, we start from Newton's approach and end with Leibnitz's approach to write the final result of derivative and antiderivative respectively. The study is strictly restricted to find the relevance of the Vedic sutras. Therefore three more formulae except sutras were added in the study to support their existence in modern mathematics especially in Calculus. **Conclusion:** Finally we have concluded that to study the derivative and antiderivative in total, three formulae for derivative and two formulae for antiderivative are sufficient i.e., using only these five formulae, we can study Calculus of single variable functions.

Key Words: Vedic Mathematics Sutra, Ekadhikena Purvena Sutra, Ekanyunena Purvena Sutra, Power Series, Derivative, and Antiderivative.

AMS Mathematics Subject Classification 2020: 20A09, 26A24, 26A36, 41A58

Introduction

A book on the Vedic Mathematics was first published in 1965 that was propounded by Bharati Krsna Tirthaji Maharaja, which is known for short-cut methods to find calculations in mathematics. These sutras have applications in many branches of mathematics like Algebra, Arithmetic, Geometry, Calculus, Trigonometry, etc.[1] Vedic mathematics is mostly based on sixteen sutras and thirteen sub-sutras. Two of them known as Ekanyunena Purvena and Ekadhikena Purvena are useful in finding derivative and antiderivative of power function of the form x^n. The meaning of 'Ekanyunena Purvena' is 'one less than the previous one' and of 'Ekadhikena Purvena' is 'one more than the previous one'. [2, 3, 4, 5, 6, 7, 8] These two sutras play an important role in Calculus, in which we study about derivative and antiderivative with their applications. Although many other mathematicians contributed in the development of Calculus, the major and equal credit goes to Isaac Newton and Gottfried Leibniz. [9, 10] It is said that the idea of differential calculus existed in India in about 6^{th} century AD, as the idea of instantaneous velocity was first developed during the 1^{st} millennium BC and Indian could solve first order differential equations as early as 6^{th} century AD, which was not possible without differential calculus concept. [11]

One of Newton's central ideas was of power series to represent functions. He provided the power series of *arcsinx* and then for *sinx* by taking the inverse of the series and so on. The power series of *sine*, *cosine*, and

arctangent had been developed in India probably in 14th century. [12] The idea of calculus existed in India in Vedic period. In Sulva-Sutra, it is seen that derivative and antiderivative existed in India. Tirthaji re-established these sutras in the form of mathematics formulae. [7, 13] In India it leaped to an amazing height in the analytic trigonometry of the Kerala School in the 14th century. The study of infinitesimal changes led to the discovery of basic principles of calculus by the time of Bhaskaracharya (1150 AD). [1, 16]
There is a possibility on the transmission of ideas of calculus from India to western countries during colonial rule in India, especially from the 16th century onwards, as there is an existence of corridor of communication between Kerala and Europe. Taylor's series expansion known as the heart of the calculus existed in India, which preceded Newton and Leibniz by centuries. [14, 15, 16, 17] Madhavacharya (1340-1425) is known for his power series expansions of different trigonometric functions and their proof presented in *Yuktibhasha* involves the idea of integration as the limit of a sum and corresponds to the algorithm of expansion and term-by-term integration. These were also found in the works of Neelkantha, Jyeshtadeva, etc. [1, 16, 17, 18]

Preliminary Ideas
To understand the importance of the two sutras, we should have the following basic concepts of Calculus:
***Power Series*:** If the power series of a function f(x) has a radius of convergence $R > 0$ and an interval of convergence $x_0 - R < x < x_0 + R$, then the series may be differentiated and integrated term-by-term i.e., once a function is written in power series, it can be differentiated and integrated quite easily, by treating every term separately. We pick up a constant of integration C, that is outside of the series here in the antiderivative of f(x). [10, 19, 20, 21]

***Power Rule of Derivative*:** If n is any integer, then

$$\frac{dx^n}{dx} = nx^{n-1}$$

This rule is valid for all real numbers. [10]
***Derivative Using Vedic Sutra*:** The derivative of power forms such as x^{10}, x^{100}, x^{1024} …etc. can be obtained by applying the Ekanyunena Purven Sutra. It says that in order to differentiate any variable x in power, multiply the index with the variable x and lower the index by 1. [6, 13] i.e.,

$$\frac{d(x^n)}{dx} = nx^{n-1} \qquad \text{(i)}$$

For example,

$$\frac{d(x^5)}{dx} = 5x^{5-1} = 5x^4$$

This can be further differentiated in a similar way to obtain second, third, …, order derivatives. The differentiation of the constant term is zero as

$$\frac{d(5)}{dx} = \frac{d(5x^0)}{dx} = 5.0.x^{0-1} = 0$$

***Power Rule of Antiderivative*:** If n (≠ -1) is any real number, then

$$\int \mathrm{x}^{\mathrm{n}}\,\mathrm{dx} = \frac{\mathrm{x}^{\mathrm{n}+1}}{\mathrm{n}+1} + \mathrm{C}$$

where C is an arbitrary constant of integration, which is generally added after finding antiderivative. This exists due to the following derivative formula

$$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$$

Here n = -1 will lead to division by zero, so it is excluded from the formula. [10]

***Antiderivative Using Vedic Sutra*:** The Vedic sutra 'Ekadhikena Purvena' gives the antiderivative of a function, which contains only powers of x. In finding integration, we use it which means "one more than the previous index to integrate the power term of the function and divide it by coefficient by the new index" [7, 13] i.e.,

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} \qquad \text{(ii)}$$

It is clear that for n = -1, division by zero occurs, which fails the sutra. Therefore we use the above rule for all real numbers except n = -1.

Example-1: Find the indefinite integral

$$\int x^7 \, dx$$

We have using the above rule

$$\int x^7 \, dx = \frac{x^{7+1}}{7+1} = \frac{x^8}{8}$$

Here Ekadhika of index 7 is 7 + 1 = 8 and the coefficient = 8. In it we may add a constant of integration.

Methodology

We know that there are two distinct approaches to integration. *Newton* followed infinite series solutions and evaluated integrals by expressing function in power series using term-by-term integration. Whereas, *Leibniz* accepted solutions in finite terms and worked for closed form expressions for integrals. Mathematicians adopted both preferences for representations of antiderivatives and later it was found that both give the same result. In the present paper we shall use first Newton's approach and then will convert the result in Leibniz form to justify that "the three rules of derivative and the two rules of antiderivative" of power series is sufficient to study the whole derivatives and antiderivative of elementary functions.

As far as the convergences of the series are concerned, we shall omit this in the present study as we know that all elementary functions in series are convergent in their domain and even if we encounter a series, which is not convergent, using the concept of the truncation of terms, we can restrict the number of terms as per our requirement of accuracy of results to make it convergent using Taylor's series (or Maclaurin's series). [10]

Discussion

The power rule of derivative is valid for all real numbers n, using it we can find the derivative of the terms like x^n and $(x \pm a)^n$. But there are functions like ln(x), which cannot be expanded in power series using Maclaurin's theorem. Also the power rule of antiderivative is valid for all real n except n = -1 i.e., using it we can find the antiderivative of the terms x^n and $(x \pm a)^n$. In other words we cannot find the antiderivative of $\frac{1}{x}$ and $\frac{1}{x \pm a}$ using power rule antiderivative. Thus we shall face problem in finding antiderivative, when a term of the form x^n or $(x \pm a)^n$comes for n = -1 in series. So first we solve these problems.

From Calculus we know that

$$\frac{d \ln(x)}{dx} = \frac{1}{x} = x^{-1} \qquad \text{(iii)}$$

Taking its antiderivative, we get

$$\int x^{-1} dx = \int \frac{dx}{x} = \ln(x) \qquad \text{(iv)}$$

We shall face one more problem, when we get the term x^n multiplied by a constant c like cx^n. So another derivative formula is needed for the function having a constant as its coefficient like cf(x). For this we shall use the linear property of derivative

$$\frac{d \, cf(x)}{dx} = c \frac{d \, f(x)}{dx} \qquad \text{(v)}$$

Using the three rules of derivative (i, iii, v) and two rules of antiderivative (ii, iv), we can find derivative and antiderivative of all most all elementary functions using term by term derivative and antiderivative.

But these are possible only if the given elementary functions are expressible in power series using Maclaurin's theorem or Taylor's theorem for some particular functions.
For verification let us find derivatives and antiderivatives of some elementary functions using Ekanyunena and Ekadhikena Purvena Sutras on each term of their power series as follows:

$$\frac{dK}{dx}=\frac{dKx^0}{dx}=K\frac{dx^0}{dx}=K.0.x^{0-1}=0$$

$$\frac{dx}{dx}=1.x^{1-1}=1.x^0=1$$

$$\frac{dsinx}{dx}=\frac{d}{dx}\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots+\frac{x^{2n-1}}{(2n-1)!}+O[x]^{2n+1}\right),n\geq 1$$

$$=1-\frac{3x^2}{3!}+\frac{5x^4}{5!}-\frac{7x^6}{7!}+\cdots+\frac{(2n-1)x^{2n-2}}{(2n-1)!}+O[x]^{2n}$$

$$=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots+\frac{x^{2(n-1)}}{(2n-2)!}+O[x]^{2n}$$

$$=\cos x,n\geq 1$$

$$\frac{dcosx}{dx}=\frac{d}{dx}\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots+\frac{x^{2n}}{(2n)!}+O[x]^{2n+2}\right),n\geq 0$$

$$=0-\frac{2x^1}{2!}+\frac{4x^3}{4!}-\frac{6x^5}{6!}+\cdots+\frac{2nx^{2n-1}}{(2n)!}+O[x]^{2n+1}$$

$$=-x+\frac{x^3}{3!}-\frac{x^5}{5!}+\cdots+\frac{x^{2n-1}}{(2n-1)!}+O[x]^{2n+1}$$

$$=-\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots+\frac{x^{2n-1}}{(2n-1)!}+O[x]^{2n+1}\right)$$

$$=-\sin x,n\geq 1$$

$$\frac{dtanx}{dx}=\frac{d}{dx}\sum_{n=0}^{\infty}(-1)^n\,T_{2n+1}\frac{x^{2n+1}}{(2n+1)!},\text{for }|x|<\frac{\pi}{2}$$

$$=\sum_{n=0}^{\infty}(-1)^n\,T_{2n+1}\frac{d}{dx}\frac{x^{2n+1}}{(2n+1)!}=\sum_{n=0}^{\infty}(-1)^n\,T_{2n+1}\frac{x^{2n}}{(2n)!}$$

$$=(-1)^0T_1\frac{x^0}{0!}+(-1)^1T_3\frac{x^2}{2!}+(-1)^2T_5\frac{x^4}{4!}+(-1)^3T_7\frac{x^6}{6!}+O[x]^8$$

$$=1+(-1)(-2)\frac{x^2}{2!}+(-1)^2(16)\frac{x^4}{4!}+(-1)^3(-272)\frac{x^6}{6!}+O[x]^8$$

$$=1+x^2+\frac{2}{3}x^4+\frac{17}{45}x^6+O[x]^8=\sec^2 x$$

In above series, T_{2n+1} is known as tangent numbers. This can also be verified using the general series expansion as

$$\frac{dtanx}{dx}=\frac{d}{dx}\left(x+\frac{x^3}{3}+\frac{2x^5}{15}+\frac{17x^7}{315}+\frac{62x^9}{2835}+O[x]^{11}\right)$$

$$=1+x^2+\frac{2x^4}{3}+\frac{119\,x^6}{315}+\frac{558\,x^8}{2835}+O[x]^{10}$$

$$=1+x^2+\frac{2x^4}{3}+\frac{17\,x^6}{45}+\frac{558\,x^8}{2835}+O[x]^{10}=\sec^2 x$$

$$\frac{dsinhx}{dx}=\frac{d}{dx}\left(\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!}\right),\text{for }|x|<\infty$$

$$=\sum_{n=0}^{\infty}\frac{d}{dx}\frac{x^{2n+1}}{(2n+1)!}=\sum_{n=0}^{\infty}\frac{(2n+1)\,x^{2n}}{(2n+1)!}=\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}=\cosh x$$

$$\frac{dcoshx}{dx}=\frac{d}{dx}\left(\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}\right),\text{for }|x|<\infty$$

$$= \sum_{n=0}^{\infty} \frac{d}{dx} \frac{x^{2n}}{(2n)!} = \sum_{n=1}^{\infty} \frac{(2n)\, x^{2n-1}}{(2n)!} = \sum_{n=1}^{\infty} \frac{x^{2n-1}}{(2n-1)!}$$

$$= \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} = \sinh x$$

$$\frac{d\tanh x}{dx} = \frac{d}{dx}\left(\sum_{n=0}^{\infty} T_{2n+1} \frac{x^{2n+1}}{(2n+1)!}\right), \text{for } |x| < \frac{\pi}{2}$$

$$= \frac{d}{dx} \sum_{n=0}^{\infty} T_{2n+1} \frac{x^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} T_{2n+1} \frac{d}{dx} \frac{x^{2n+1}}{(2n+1)!}$$

$$= \sum_{n=0}^{\infty} T_{2n+1} \frac{(2n+1)x^{2n}}{(2n+1)!} = \mathrm{sech}^2 x$$

$$\frac{de^x}{dx} = \frac{d}{dx}\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} + O[x]^{n+1}\right)$$

$$= 0 + 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \frac{4x^3}{4!} + \cdots + \frac{nx^{n-1}}{n!} + O[x]^n$$

$$= 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^{n-1}}{(n-1)!} + O[x]^n = e^x$$

etc. Similarly we can find the antiderivative using Vedic sutra as follows

$$\int \cos x \, dx = \int \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + \frac{x^{2n}}{(2n)!} + O[x]^{2n+2}\right) dx$$

$$= x - \frac{x^3}{2!\,3} + \frac{x^5}{4!\,5} - \frac{x^7}{6!\,7} + \cdots + \frac{x^{2n+1}}{(2n)!\,(2n+1)} + O[x]^{2n+3} + K$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + O[x]^{2n+3} + K$$

$$= \sin x + K$$

where K is a constant of integration.

$$\int \sin x \, dx = \int \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + O[x]^{2n+3}\right) dx$$

$$= \frac{x^2}{2} - \frac{x^4}{3!\,4} + \frac{x^6}{5!\,6} - \frac{x^8}{7!\,8} + \cdots + \frac{x^{2n+2}}{(2n+1)!\,(2n+2)} + O[x]^{2n+4} + K$$

$$= -1 + \frac{x^2}{2} - \frac{x^4}{3!\,4} + \frac{x^6}{5!\,6} - \frac{x^8}{7!\,8} + \cdots + \frac{x^{2n+2}}{(2n+1)!\,(2n+2)} + O[x]^{2n+4} + (K+1)$$

$$= -\left(1 - \frac{x^2}{2} + \frac{x^4}{3!\,4} - \frac{x^6}{5!\,6} + \frac{x^8}{7!\,8} - \cdots + \frac{x^{2n+2}}{(2n+1)!\,(2n+2)} + O[x]^{2n+4}\right) + (K+1)$$

$$= -\cos x + C$$

where C = K + 1 is a constant of integration.

$$\int \cosh x \, dx = \int \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} dx = \sum_{n=0}^{\infty} \int \frac{x^{2n}}{(2n)!} dx = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)(2n)!}$$

$$= \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} = \sinh x$$

$$\int \sinh x \, dx = \int \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} dx = \sum_{n=0}^{\infty} \int \frac{x^{2n+1}}{(2n+1)!} dx$$

$$= \sum_{n=0}^{\infty} \frac{x^{2n+2}}{(2n+2)(2n+1)!} = \sum_{n=0}^{\infty} \frac{x^{2(n+1)}}{(2n+2)!} = \sinh x + K$$

$$\int e^x \, dx = \int \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} + O[x]^{n+1}\right) dx$$

$$= x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^{n+1}}{(n+1)!} + O[x]^{n+2}$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^{n+1}}{(n+1)!} + O[x]^{n+2} - 1 + k$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^{n+1}}{(n+1)!} + O[x]^{n+2} + K = e^x + K$$

Following the same procedures and the following expansion of other functions in series as

$$x \cot x = 1 - \sum_{n=1}^{\infty} (-1)^{n-1} 2^{2n} B_{2n} \frac{x^{2n}}{(2n)!}, \text{for } |x| < \pi$$

$$\sec x = \sum_{n=0}^{\infty} (-1)^{n} E_{2n} \frac{x^{2n}}{(2n)!}, \text{for } |x| < \frac{\pi}{2}$$

$$x \csc x = 1 + \sum_{n=1}^{\infty} (-1)^{n-1} 2(2^{2n-1} - 1) B_{2n} \frac{x^{2n}}{(2n)!}, \text{for } |x| < \pi$$

$$x \coth x = 1 + \sum_{n=1}^{\infty} 2^{2n} B_{2n} \frac{x^{2n}}{(2n)!}, \text{for } |x| < \pi$$

$$\operatorname{sech} x = \sum_{n=0}^{\infty} E_{2n} \frac{x^{2n}}{(2n)!}, \text{for } |x| < \frac{\pi}{2}$$

$$x \operatorname{csch} x = 1 - \sum_{n=1}^{\infty} 2(2^{2n-1} - 1) B_{2n} \frac{x^{2n}}{(2n)!}, \text{for } |x| < \pi$$

etc. we can verify their derivatives and antiderivatives using ekanyunena and ekadhikena purvena sutras. [10, 22, 23, 24] In the above expansions B_{2n} and E_{2n} are called Bernoulli numbers and Euler numbers respectively. The values for the tangent numbers, the Bernoulli numbers and the Euler numbers are as follows for $n \leq 8$ in the following table-1: [22, 24]

Table-1

Natural Numbers	Tangent Numbers	Bernoulli Numbers	Euler Numbers
n	T_{2n+1}	B_{2n}	E_{2n}
0	1	1	1
1	-2	1/6	-1
2	16	-1/30	5
3	-272	1/42	-61
4	7936	-1/30	1385
5	-353792	5/66	-50521
6	22368256	-691/2730	2702765
7	-1903757312	7/6	-199360981
8	209865342976	-3617/510	19391512145

For those functions, whose expansion is not so easy to remember, their power series representation can be found using Mathematica as:

Input Function	Output Power Series
In[1]: Series[Tan[x], {x, 0,10}]	Out[1]: $x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + O[x]^{11}$
In[2]: Series[Sec[x], {x, 0,10}]	Out[2]: $1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \frac{277x^8}{8064} + \frac{50521x^{10}}{3628800} + O[x]^{11}$
In[2]: Series[Sec$^{(2)}$[x], {x, 0, 5}]	Out[3]: $1 + x^2 + \frac{2}{3}x^4 + \frac{17}{45}x^6 + O[x]^8$

Limitations

We stated earlier that mathematicians expressed different preferences finite vs. infinite series for representations of derivatives and antiderivatives. So there is no harm to follow either one but the problem starts when after operations applied, the new series is not an expression of any known elementary functions. Because every finite or infinite series can't be a series expansion of an elementary function neither can be expressed in closed form of elementary functions. For example,

$$\int \frac{e^x}{x}\mathrm{dx} = \int \frac{1}{x}\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^{\mathrm{k}}}{\mathrm{k}!} + \mathrm{O}[\mathrm{x}]^{\mathrm{k}+1}\right)\mathrm{dx}$$

$$= \int \left(\frac{1}{x} + 1 + \frac{x}{2!} + \frac{x^2}{3!} + \cdots + \frac{x^{\mathrm{k}-1}}{\mathrm{k}!} + \mathrm{O}[\mathrm{x}]^{\mathrm{k}}\right)\mathrm{dx}$$

$$= \ln(\mathrm{x}) + x + \frac{x^2}{2!\,2} + \frac{x^3}{3!\,3} + \cdots + \frac{x^{\mathrm{k}}}{\mathrm{k}!\,\mathrm{k}} + \mathrm{O}[\mathrm{x}]^{\mathrm{k}+1} + \mathrm{K}$$

This series can't be denoted by an elementary function. That's why it is known as a nonelementary function or nonelementary integral. [26] Thus term-wise derivative and antiderivative terminate at this point. Modern techniques of integration also don't solve these problems.

Conclusion

Following the above results we conclude that to study the derivative of elementary functions, following three derivative formulae

$$\frac{d}{dx}x^n = nx^{n-1}, \frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)], \frac{d}{dx}\ln(x) = \frac{1}{x}$$

are sufficient and to study the antiderivative of elementary functions following two formulae

$$\int x^n\, dx = \frac{x^{n+1}}{n+1}, \int \frac{1}{x}dx = \ln(x).$$

are sufficient. In these five formulae, the Vedic Mathematics Ekanyunena Purvena and Ekadhikena Purvena Sutras play an important role. Perhaps this might have been the prime reason that ancient Indian mathematicians didn't think for another rules of derivative and antiderivative as per the requirement in then and at that time related to solving techniques of real world problems.

Future Scope of Research

The limitation of the lack of notation of the functions creates opportunity for new research. As has been discussed in the limitations section that mathematics world lacks notations of many infinite series. Which indicate that the scope of research is available for new functions originated from nonelementary integrals.

References

1. Tekriwal, G. Maths Sutras (From Around the World), Penguin Random House India Pvt. Ltd, Haryana, 2017, xi-xiv.
2. Thakur, R. The Essentials of Vedic Mathematics, Rupa Publication India Pvt. Ltd, New Delhi, 2013, xi-xxi.
3. Tirthaji, B. K. Vedic Mathematics, Motilal Banarsidass Publishers Private Limited, New Delhi, 2015, v-xii, 185-187.
4. Wikipedia contributors. Calculus. Wikipedia, The Free Encyclopedia. March 11, 2023, 17:45 UTC. Accessed April 4, 2023.
5. Anton, H., Bivens, I., & Davis, S. Calculus, 7th Edition, Wiley, 2014, 10-13, 185-186, 191, 196, 317, 323-326, 693-716.
6. Girish, T. E. & Nair, C. R. Ideas of Physical Forces and Differential Calculus in Ancient India. n.d.
7. Katz, V. J. Ideas of Calculus in Islam and India, Mathematics Magazine, 1995, 68 (3): 163-174, June.

8. Garain, D. N. & Kumar, S. Analytical Discussion for Existence of Idea of Calculus in Vedic and Post Vedic Periods, International Journal of Mathematics Trends and Technology (IJMTT), 2018, 61(2): 85-88.
9. Almeida, D. F. & Joseph, G. G. Kerala Mathematics and Its Possible Transmission to Europe. n.d.
10. Almeida, D. F. & Joseph, G. G. Eurocentrism in the History of Mathematics: the Case of the Kerala School, Race Class, 2004, 45(4): 45-59.
11. Divakaran, P. P. Calculus in India: The Historical and Mathematical Context, Current Science (Supplement), 2010, 99(3): 8-14, 10 August.
12. Osgood, W. F. Non-uniform convergence and the integration of series term by term. *American Journal of Mathematics*, 1897, *19*(2): 155-190.
13. Post, E. L. Generalized differentiation. *Transactions of the American Mathematical Society*, 1930, *32*(4): 723-781.
14. Abramowitz, M. & Stegun, I. A. Handbook of Mathematical Functions, Dover Publications, Inc., New York. 1965.
15. Roy, R. Series and Products in the Development of Mathematics. 2021, 1(2nd ed.). Cambridge University Press.
16. Wikipedia contributors. Taylor series. Wikipedia, The Free Encyclopedia. March 17, 2023, 09:09 UTC. Accessed April 4, 2023.
17. Wikipedia contributors. Nonelementary integral. Wikipedia, The Free Encyclopedia. August 13, 2022, 00:04 UTC. Accessed April 4, 2023.
18. Yadav, D. K. Nonelementary Integrals: Indefinite Nonintegrable Functions, Notion Press, India, pp.19, 22, 23, 27, 2023.