

Study of Rheological Properties in Human Blood Cells in Narrow Capillaries Using Tank Treading Method

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Abstract: In this study we discussed the rheological properties in human blood cells through narrow capillaries using tank treading method. Here the behavior of low condensed flow of fluid of fluid has been analyzed and it is obtained that the shear flow of red cells may flip on the shear rate and structure of the blood cells. The case of tank-treading without flipping the corpuscles strain is anticipated while a task of the shearing rate and the aligned angle. In the instance of roll over, the low condensed of blood corpuscles represents the Newtonian fluid manner whereas condensed blood flow represents non-Newtonian fluid manner. Also, tank treading motions of blood corpuscles on the rheological properties are also studied in this paper.

Keywords: Rheological Properties, Blood flow, shear thinning

1. Introduction

In the human body, arteries carry blood away from the heart and veins carry blood back to the heart. The circulatory system carries oxygen, nutrients and hormones to cells, and removes waste products, like carbon dioxide. These roadways travel in one direction only, to keep things going where they should. When the shear rate is low, the red blood cells form rouleaux by accumulation is represented in figure1. This figure is represented by Fung (1981). As the shear rate is increased, the average number of red blood cells in each rouleaux has been decreased. If shear rate is larger than a certain standard value, the rouleaux has been broken up into individual cells. Schmidt-Schonbein et al. (1969) obtained the influence of deformability of human cells using *vitro* experiments on the blood flow at low concentrations of red blood cells (5-9.5%).

Normal cells has observed to be separate, not in rouleaux, at shear rate $k = 4.5 \text{ s}^{-1}$. It is observed that due to slight leads of k , the individual cells were found accompanied by periodic fall down and orbiting in flow. As k was further leads, the cells flatter orientate and lack orbiting has been seen. For $k > 100 \text{ s}^{-1}$, the separate blood cells forgotten his biconcave structure and changed to a variety of structures, numerous be similar to prolate egg-shaped with major axis equidistant to direction of flow with no fall. Richardson (1974) has studied the deformation and haemolysis of the red blood cells in shearing flow of fluid.

Goldsmith (1971) has analyzed the very close monitoring from the Poiseuille blood flow investigations. In his investigation he proposed that at small shear rate, the blood cells move but as flow rates up, revolving is no longer investigated, i.e., the structure of cells made motionless in comparison to axes fixed in space. These types of movement are called the tank-treading motion. The aim of this study is to analyze the rheological property of human blood cells in narrow capillaries using tank treading method. The above characteristics of blood cells are considered to predict the effective viscosity of blood as a function of shear rate. Singh and Trivedi (2017) have studied analysis of blood flow through narrow tapered tubes. In another paper Singh et al (2022) have investigated magnetic field effects on oscillatory couette flow regime.

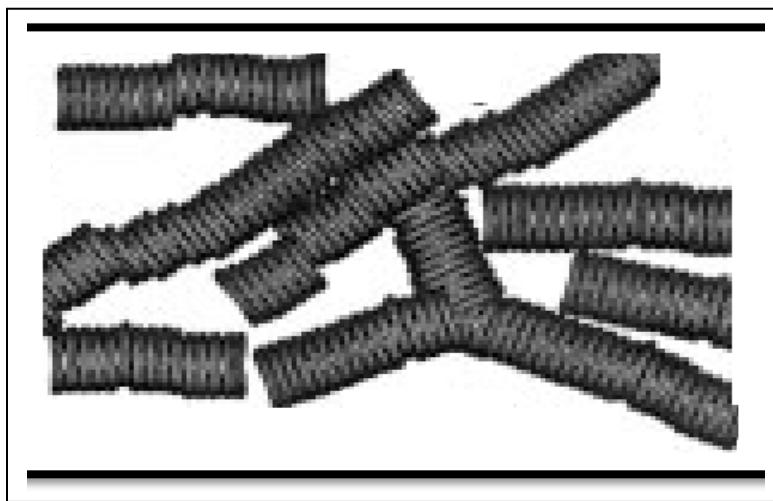


Fig.1. Shape of Human Blood cell

2. Behavior of Blood Flow in Human Body

The red blood cells are flexible during the movement of arteries and veins, because the diameter of blood cells is large that the diameter of very small arteries and veins. The red blood cells during movement of it, adapt easily to their immediate environment and exert no force on the cells. This behavior is determined by elastic properties of red cells membrane which must be carefully taken in consideration when creating the

computational model of red blood cells. In the typical human, the red blood cell is a disk diameter of approximately 6.1 to 8.3 Micrometer and the thickness at the thickest point of 2 to 2.6 micrometer and the minimum thickness is the centre of 0.8 to 1 micrometer, being much smaller than most other human cells. Srivastava (2003) has studied flow of a couple stress fluids representing blood through stenotic vessels with a peripheral layer. Mishra and Panda (2005) has studied Newtonian model for blood flow through an arterial stenosis. The Pulsating flow of a hydromagnetic flow between permeable beds has been studied by Malathy and Srinivas (2008). A micro polar fluid model of blood flow through a tapered artery with a stenosis has been studied by Abdullah and Amin (2010).

Adult humans have roughly 20 to 30 trillion red blood cells at any given time, constituting approximately 70% of all cells by number. Women have about 4-5 million red blood cells per micro liter of blood and men about 5-6 million. The people living at high altitudes with low oxygen tension will have more blood. Red blood cells are thus much more common than the other blood particles: there are about 4000 – 11000 white blood cells and about 150000 to 400000 platelets per micro liter. Human red blood cells take on average 60 seconds to complete one cycle of circulation.

The blood's red color is due to the spectral properties of the heme iron that is ions in hemoglobin. Each hemoglobin molecule carries four heme groups: hemoglobin constitutes about a third of the total cells volume. The hemoglobin is responsible for the transport of more than 98% of the oxygen in the body. The red blood cells of an average adult human male store collectively about 2.6 grams of iron, representing about 65% of the total iron contained in the body. Chaubey et al (2012) have studied the flow of closely fitting elastic particles in very narrow vessels. Kutev et. al. (2015) have studied approximation of the oscillatory blood flow using the Carreau viscosity model. Qasim et. al. (2019) have studied numerical simulation of MHD peristaltic flow with variable electrical conductivity and Joule-dissipation using generalized differential quadrature method. Yadav et. al. (2023) has studied the investigations of blood flow through stenosed vessel using non-Newtonian micro polar fluid model. Singh et al. (2024) presented hall effects on thermal oscillatory boundary layer flow of fluid.

3. Discussions and Results of the effective properties in the high shear rates
In this analysis we assumed the case of higher shearing rate flow of blood in human body. Here the shearing rate is sufficiently higher for the cells to exist separately in plasma. The effectual characteristics of blood flow are obtained mostly by the deformation and motion of separate blood cells. In this part, we assume the following:
(i).Axisymmetric straining flows in aligned cells, and (ii). Motion of tank-treading in shearing flows

3.1. Aligned Cells in Axisymmetric Straining Flows in Aligned Cells

Let us assume the easiest situations for these separate cells that are totally aligned due to the axi-symmetric straining flow of fluids. The red blood cells are easily flexible and change its shape. It is deformable in straining flow of blood and seen inspheroidal

equilibrium situations in axi-symmetric straining flows if the layer tension is considered to be isotropic.

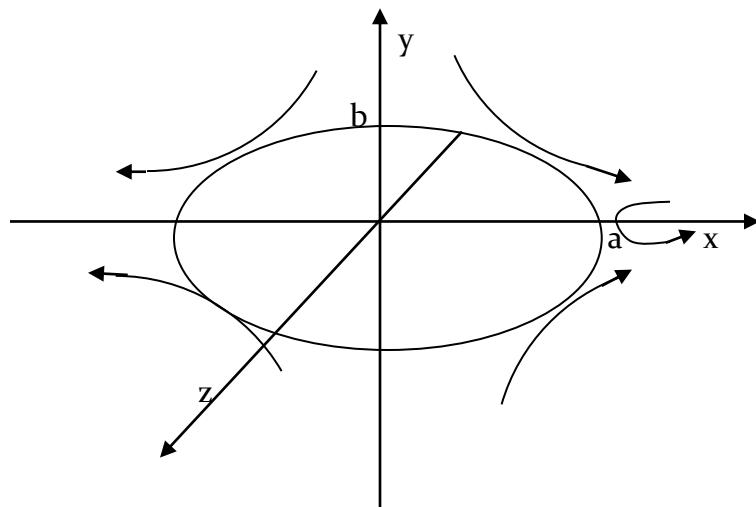


Fig.2a.Egg-Shaped blood cells flow in uniaxial and biaxial

Since the cells deformity should convince the simultaneous demand of capacity conservation, the area maintaining the nature of cells deformity is different from that of the deformable corpuscles or small part of fluid. Differently from the condition of flexible particles, the equilibrium structure is not dependent of the rate of strain, if the curving resistance of blood cells is abandoned. On the other hand, it is obtained that the tension of the membrane leads as the lead of strain rate. In these parts, the effect of shape-conserving belongings on the suspension Rheology is studied for two axi-symmetric straining flows. In uniaxial flow, straining flow of each blood cell is changed into Egg -shaped structure and in a biaxial flows, straining flow is changed into elliptical structure, which is given above fig 2a, 2b.

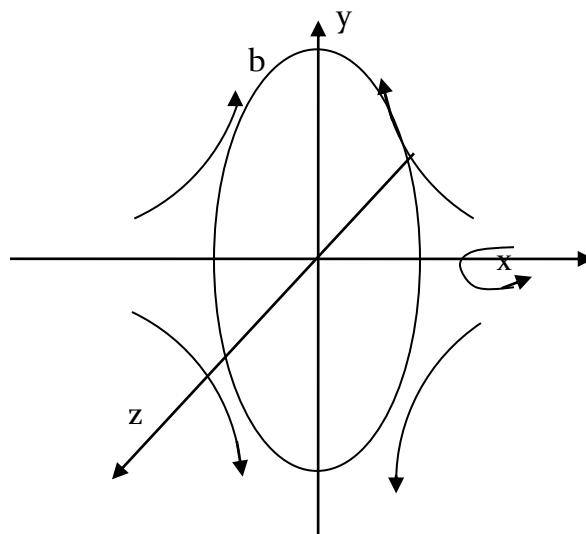


Fig.2b. Elliptical blood cells flow in uniaxial and biaxial

Principal variable for the distortion of blood cells is the sphericity pointer Δ represented by

$$\Delta = \frac{(A/4\pi)^{\frac{1}{2}}}{(3V/4\pi)^{\frac{1}{3}}} \quad (1)$$

where A represents the total surface area and V stand for the volume of the cell.

For the situation of round corpuscles, index is a function of the aspect ratio $R = a/b$. For the spheroidal clue of human red blood cells, it is able to accept that the area of a red blood cell is nearly 43.55% larger than minimum area acquired for the ball shape structure. Then by the definition of the index we have $\Delta = 1.2$ and we obtained

$$R = 0.255 \text{ for an elliptical structure,} \quad (2)$$

$$R = 6 \text{ for an egg-shaped structure,} \quad (3)$$

Let we discuss the situation of uni-axial straining blood flow in which red blood cells take an Egg-shaped equilibrium structure. When the aspect ratio $R = 6$, the coefficients is estimated by Hinch and Leal (1972)

$$A_{11} = 9.1281, B_{11} = 0.05390, C_{11} = 1.999 \text{ (assume as 2)} \quad (4)$$

$$\chi^{(p)} = 2v\varphi \left\{ 18.2691 E : \langle pppp \rangle + 0.10792 \left(E : \langle pp \rangle + \langle pp \rangle : E - \frac{2}{3} I E : \langle pp \rangle \right) + 2E \right\}, \quad (5)$$

In the situation of uniaxial blood flow, all cells are considered to be aligned in the e_x -direction and thus

$$p = e_x; \quad \mathbb{E} = E \left[e_x e_x - \frac{1}{2} e_y e_y - \frac{1}{2} e_z e_z \right], \quad (6)$$

where \mathbb{E} is main strain rate, E is rate of strain tensor, putting the above equation into (5) and subtracting the isotropic, we get the result

$$\chi^{(p)} = 2v\varphi(14.32868)E, \quad (7)$$

Or we have

$$v^* = v(1 + 14.328\varphi), \quad (8)$$

In part of biaxial straining flow, the aspect ratio of equality cells structure is $r = 0.255$ and the coefficients are obtained as

$$A_{11} = 2.2919, B_{11} = -1.91696, C_{11} = 4.8197, \quad (9)$$

But substituting the above coefficients, $p = e_x$ and

$$\mathbb{E} = E \left[-e_x e_x + \frac{1}{2} e_y e_y + \frac{1}{2} e_z e_z \right], \quad (10)$$

We has to subtract the isotropic equation, we get the result given below

$$\chi^{(p)} = 2v\varphi(2.7778)E, \quad (11)$$

The effective viscosity for the biaxial straining flow is given by

$$v^* = v(1 + 2.7778\varphi), \quad (12)$$

The effectual viscosity for a low concentrated of flexible corpuscles was obtained by Cho (1992) under the consideration that flexibility from spherical structure not huge. The output is given by

$$\chi^{(p)} = 2v\varphi \left\{ \frac{5}{2} \pm 2.677848 + 15.38555 \delta^2 \right\} E; \quad \delta = \frac{vE}{G}, \quad (13)$$

Here \pm represents to the uniaxial and biaxial straining fluid flow comparatively, and E and G are the main strain rate and shear modulus of the flexible corpuscles.

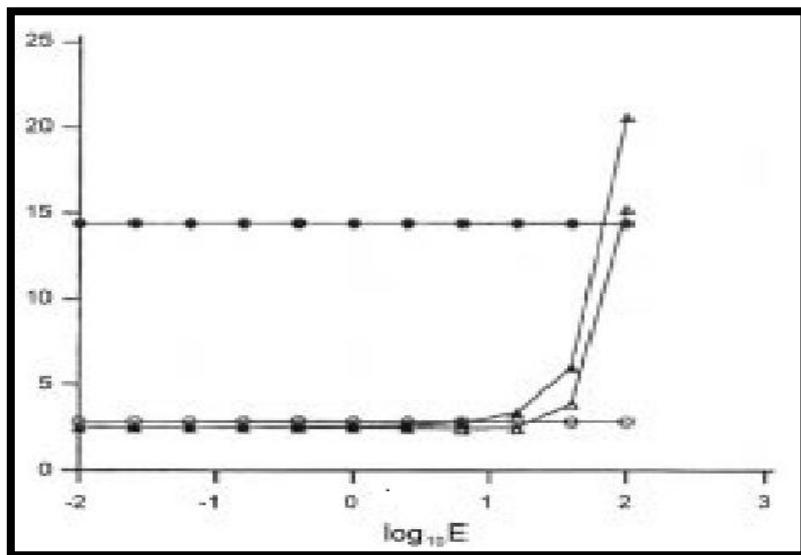


Fig.3. Graph Viscosity factor $(v^* - v)/\varphi$ as a function flow (Red blood cells in a biaxial flow)

The effectual viscosity corresponding to relations (12) is also presented below

$$v^* = v \left[1 + \left(\frac{5}{2} \pm 2.677848 + 15.38555 \delta^2 \right) \varphi \right], \quad \delta = \frac{vE}{G}, \quad (14)$$

The working viscosity factor $(v^* - v)/\varphi$ is presented in Fig. 3. The closed and open round are for the working viscosity or effectual viscosity in human blood cells in the situation of uniaxial and biaxial straining flows individually is presented here. The closed and open triangles are for effective viscosity of suspension of flexible particles in the cases of uniaxial and biaxial straining flows. For the situation of flexible corpuscles, $v/G = 0.01s$ is taken in plot of graph.

3.2. Motion of Tank-treading in Shearing Flows

Now we have obtained the bulk stress of human blood. The assumption of human blood subject to shear flow is considered to assume the tank-treading flow, which is represented in above Fig. 4. On the other hand to grow a hypothesis on the effectual

viscosity, Batchelor's (1970) hypotheses for the low concentrated of ellipsoidal rigid particles is changed appropriately to assume the tank-treading movement of RBC. Then adapted statement is contained with the postulation of Kellar and Skalakin tank-treading movement of RBC to find the rheological property of human blood flow in arteries and veins.

3.2.1. Distraction Area of Tank-Treading for RBC

The assumption of tank-trading method shape of RBC in ellipsoidal form is represented in Fig. 4. In this method x_i represents the coordinate of a different Cartesian coordinate structure and \bar{x}_i represents the coordinates in the second Cartesian coordinate structure. The origin has coincided with the stable shape. The x_3 axis is consider to coincide through \bar{x}_3 axis, but \bar{x}_1 and \bar{x}_2 axis are revolve with angle θ with regard to x_1 and x_2 axis. The elliptical plane shape is described by the semi-axes a, b , and c on \bar{x}_1, \bar{x}_2 , and \bar{x}_3 , corresponding. The diaphragm facet velocity represented by v_i^m comparative to bodywork setting is considered as

$$\bar{v}_1^m = \eta(-a/b)\bar{x}_2, \quad \bar{v}_2^m = \eta(b/a)\bar{x}_1, \quad \bar{v}_3^m = 0, \quad (15)$$

here η represents the parameter possess the width of pulsation,

For an elliptical corpuscles with exterior velocity in the formation

$$\bar{v}_i^m = (\bar{E}_{ij}^m + \bar{\Omega}_{ij}^m)\bar{x}_j, \quad (16)$$

In the motion area which has represented by

$$\bar{U}_i = (\bar{E}_{ij} + \bar{\Omega}_{ij})\bar{x}_j, \quad (17)$$

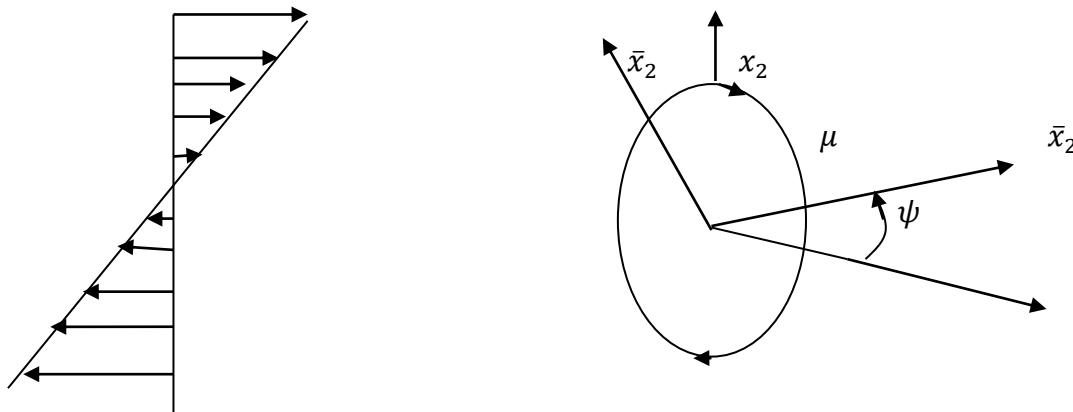


Fig.4. Ellipsoid RBC passing in tank-treading movement due to shear flow

A long away beginning the corpuscles, Roscoe (1967) presented that the disruption is the identical as might be made by a stiff, non-revolving elliptical in a fluids go through the continuous flow of fluid

$$\bar{U}_i^0 = (\bar{E}_{ij} - \bar{E}_{ij}^m) \bar{x}_j + (\bar{\Omega}_{ij} - \bar{\Omega}_{ij}^m) \bar{x}_j, \quad (18)$$

here the debar amount or magnitude are mention to working coordinate structure.

Distraction blood flow field is developed by change of strain tensor $(E_{kl} - E_{kl}^m)$.this is the part of the tensor $E - E^m$ mentioned to stable form. Hence we get

$$D_{ij} = C_{ijkl}(E_{kl} - E_{kl}^m), \quad (19)$$

For shearing flows,

$$E_{ij} = \frac{\kappa}{2} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (20)$$

The velocity of external surface layer is given by

$$\bar{E}_{ij}^m = \left(\frac{a^2-b^2}{2ab} \right) \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (21)$$

here \bar{E}_{ij}^m is represented to \bar{x}_i coordinates. In order to transform \bar{E}_{ij}^m to E_{ij}^m , we apply the relation

$$E_{ij}^m = \bar{E}_{kl}^m \gamma_{ik} \gamma_{jl}; \quad \gamma_{ik} = \bar{e}_i \cdot e_k, \quad (22)$$

where e_i and e_k are unit vectors and it is parallel to \bar{x}_i and \bar{x}_k coordinates. Then for $E_{ij} - E_{ij}^m$, we have obtained

$$E_{ij} - E_{ij}^m = \frac{\kappa}{2} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \left(\frac{a^2-b^2}{2ab} \right) \begin{pmatrix} -\sin 2\theta & \cos 2\theta & 0 \\ \cos 2\theta & \sin 2\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (23)$$

In this present situation, the main directions is given by

$$p_i = (\cos \theta, \sin \theta, 0), \quad q_i = (-\sin \theta, \cos \theta, 0), \quad r_i = (0, 0, 1), \quad (24)$$

Then we get the results

$$\frac{C_{ijkl}}{abc} (E_{kl} - E_{kl}^m) = \left(\frac{\kappa}{2} \right) \frac{4 \{ J_1 (p_i p_j - \frac{1}{3} \delta_{ij}) - J_2 (q_i q_j - \frac{1}{3} \delta_{ij}) \}}{3(J_1 J_2 + J_2 J_3 + J_3 J_1)} + \left(\frac{2}{3I_3} \right) \left(\kappa \cos 2\theta \theta + \frac{a^2-b^2}{2ab} \eta \right) (p_i q_j + p_j q_i), \quad (25)$$

3.2.2. Tank Treading and Flipping Velocity

The motion of the tank-treading ellipsoid cells is a shear flow that is given by Keller and Skalak (1982) which is shown in Fig. 4. By using the movement's equilibrium on human blood corpuscles, they have obtained the slip or roll over velocity and also obtained the tank-treading pulsation by equating the energy dissolute inside cells and it is contributed by outer fluid. Now to integrate this outcome, he obtained that the slip or rolling velocity is given by

$$\dot{\psi} = A + B \cos 2\psi, \quad (26)$$

here

$$A = \frac{\kappa}{2}, \quad B = \kappa \left[\frac{4a^2b^2}{a^4-b^4} \left\{ 2 + \left(\frac{v'}{v} - 1 \right) J_3 \right\}^{-1} + \frac{1}{2} \left(\frac{a^2-b^2}{a^2+b^2} \right) \right], \quad (27)$$

where I_3 represents the integral, which is defined by Batchelor, and is given below

$$I_3 = \int_0^\infty \frac{abc(a^2+b^2)d\lambda}{\Delta(a^2+\lambda)(b^2+\lambda)}, \quad (28)$$

Where $\Delta^2 = (a^2 + \lambda)(b^2 + \lambda)(c^2 + \lambda)$, as we may see the above, the blood flow particle go through tank-treading shifting without slipping if $0 \leq -A/B \leq 1$.

Equilibrium angle of inclination is given below

$$\psi^* = \frac{1}{2} \cos^{-1} \left(-\frac{A}{B} \right), \quad (29)$$

The associated tank-treading pulsation is given by

$$\nu^* = \frac{2ab}{a^2-b^2} \left[\left\{ 2 + \left(\frac{v'}{v} - 1 \right) I_3 \right\}^{-1} \frac{\kappa A}{B} \right] \leq 0, \quad (30)$$

In the other situation, if $B < -A$, the blood cells go through the slipping movement. The solution of (26) is

$$\psi(t) = \arctan \left[\frac{A+B}{(A^2-B^2)^{\frac{1}{2}}} \tan \left\{ \frac{(t-t_0)\pi}{T} \right\} \right], \quad (31)$$

here t_0 stands for the time at $\psi=0$, T stands the period of slipping from $\psi = 0$ and $\psi = -\pi$. Then T is represented by

$$T = \pi(A^2 - B^2)^{-\frac{1}{2}} \quad (32)$$

3.2.3. Dilute Suspension of RBC and Particle Stress

The red blood cells go through tank-treading method, without slipping. It has of homogeneous structures and direction. In this situation the blood cells particle stress is represented by

$$\chi_{i,j}^{(p)} = 3\nu \left(\frac{\chi_3^4 \pi abc}{V} \right) \left(\frac{c_{ijkl}}{abc} \right) (E_{kl} - E_{kl}^m). \quad (33)$$

Now using the information in (25), we simply show that

$$\frac{\chi_{i,j}^{(p)}}{3\nu\phi} = \frac{\kappa \sin 2\psi}{3(J_1J_2+J_2J_3+J_3J_1)} [J_1X_{ij} - J_2Y_{ij}] + \frac{3\kappa \cos 2\psi}{3I_3} \left[1 - \frac{2}{2+I_3\left(\frac{v'}{v}-1\right)} \right] Z_{ij}, \quad (34)$$

here

$$X_{ij} = \begin{pmatrix} \frac{1}{3} + \cos 2\psi & \sin 2\psi & 0 \\ \sin 2\psi & \frac{1}{3} - \cos 2\psi & 0 \\ 0 & 0 & -\frac{2}{3} \end{pmatrix}, \quad (35)$$

$$Y_{ij} = \begin{pmatrix} \frac{1}{3} - \cos 2\psi & -\sin 2\psi & 0 \\ -\sin 2\psi & \frac{1}{3} + \cos 2\psi & 0 \\ 0 & 0 & -\frac{2}{3} \end{pmatrix}, \quad (36)$$

$$Z_{ij} = \begin{pmatrix} -\sin 2\psi & \cos 2\psi & 0 \\ \cos 2\psi & \sin 2\psi & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (37)$$

here ϕ stands for the volume fraction of the red blood cells (RBC), i.e.,

$$\varphi = \chi \frac{4}{3} \pi abc / V. \quad (38)$$

Here the human red blood corpuscles go through flipping movement. The mean corpuscles stress may be acquired as follows

$$\frac{\chi_{i,j}^{(p)}}{3v\varphi} = \frac{1}{T} \int_0^T \left(\frac{\chi_{i,j}^{(p)}}{3v\varphi} \right) dt = \frac{1}{T} \int_0^{-\pi} \left(\frac{\chi_{i,j}^{(p)}}{3v\varphi} \right) \frac{d\psi}{\psi} = \frac{1}{T} \int_0^{-\pi} \left(\frac{\chi_{i,j}^{(p)}}{3v\varphi} \right) \frac{d\psi}{(A+B\cos 2\psi)}, \quad (39)$$

Now we should required note that mean time should be same as that found by the possibility of distribution. The possibility distribution is expressed by the equation given below

$$\frac{\partial p}{\partial t} + \nabla \cdot (p \dot{\psi} e_\psi) = 0. \quad (40)$$

In the steady state flow

$$\frac{\partial}{\partial \psi} (p\dot{\psi}) = 0, \text{ i.e., } p \propto \frac{1}{\dot{\psi}} \quad (41)$$

By using

$$\int_0^{-\pi} \frac{\sin^2 2\psi}{A+B\cos 2\psi} d\psi = \frac{1}{2} \frac{\pi}{(A^2-B^2)^{\frac{1}{2}}} \left[1 - \frac{B^2/A^2}{\{1+(1-B^2/A^2)^{1/2}\}^2} \right] \equiv \frac{T}{2} I_s, \quad (42)$$

and

$$\int_0^{-\pi} \frac{\cos^2 2\psi}{A+B\cos 2\psi} d\psi = \frac{1}{2} \frac{\pi}{(A^2-B^2)^{\frac{1}{2}}} \left[1 + \frac{B^2/A^2}{\{1+(1-B^2/A^2)^{1/2}\}^2} \right] \equiv \frac{T}{2} I_c, \quad (43)$$

We cansimply show that

$$\frac{\chi_{i,j}^{(p)}}{3v\varphi} = \left[\frac{(J_1+J_2)I_s}{3(J_1J_2+J_2J_3+J_3J_1)} + \frac{2}{3} \frac{I_c}{I_3} \left\{ 1 - \frac{2}{2+I_3\left(\frac{v'}{v}-1\right)} \right\} \right] E_{ij}, \quad (44)$$

here

$$E_{ij} = \begin{pmatrix} 0 & \frac{\kappa}{2} & 0 \\ \frac{\kappa}{2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (45)$$

Here we can perceive the aboveresults, the average blood particle stress is stated in the formation of Newtonian fluid and effective viscosity of fluid is presented as below

$$\frac{v^*}{v} = 1 + \left[\frac{(J_1+J_2)I_s}{2(J_1J_2+J_2J_3+J_3J_1)} + \frac{2}{3} \frac{I_c}{I_3} \left\{ 1 - \frac{2}{2+I_3\left(\frac{v'}{v}-1\right)} \right\} \right] \varphi, \quad (46)$$

In this analysis the main results is equation (34) and (46). For the assessment of output, we required the structure of tank-treading or slipping of red blood corpuscles. For this motivation, we may perceive not dependent theoretical or experimental work.

4. Conclusions

The main objective of the study was to conclude the rheological properties of human blood cells in narrow capillaries using tank-treading method. We assumed the tank-treading flow of red blood cells. In this study, the theory of Batchelor for low concentration interruption has changed that it tackled with Keller and Skalak for tank-

treading motion of red blood cells. Shearing rate of blood cells may tip above on the shear rate and the shape of RBC, etc. In the tank treading without flip, the RBC corpuscles stress is prognosticated as a work of shear change in the inclined angle. During the tipping above, the low condensed blood shows as Newtonian fluid characters whereas the more condensed blood represents as non-Newtonian fluid characters. The prognosticate procedure for the forecast viscosity taken in a required parameter, which is the structure of the cells. Here mentioned equation (46) is the main output. The variable must be acquired by an investigational work presented by Rechardson.

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