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Solution of Time Fractionalnewell–Whitehead–Segal Equation Using Modified Adomian Decomposition Method Elzaki Transformation Method

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Abstract

In this study, applied Modified Adomian Decomposition Elzaki Transformation method (MADETM) on time fractional nonlinear Newell–Whitehead–Segal (NWS) equation to obtain the series solution. Approximate solutions are quickly converging to exact solutions in numerical. The adopted technique is compared with other methods like NHPTM, FCT-HP, VIM respectively to validate the results. The graphical presentations shown with the compared methods. The error analysis &statistical analysis is performed on NWS equation by considering small sample t-test to identify the significance level. The results of hypothesis testing indicated that there is no statistically significant variation in the mean scores between the two solutions. suggesting that there is a meaningful distinction in the outcomes associated with the two conditions.

Keywords: Fractionalpartial differential equation, Newell–Whitehead–Segal (NWS) equation, Modified Adomian Decomposition Method

Introduction:

Fractional calculus (FC) is one of the branch topics in applied mathematics with several ways to determine the powers of operators, whether they be differential or integral, using real or complex numbers. Fractional differential equation is useful tool in identifying nonlinear oscillations, along the continuous traffic flow procedure, in fluid mechanics. Mathematical bio-medical field, chemistry, and numerous additional technical and physical procedures are also simulated using FDEs. Because most physical systems in nature are nonlinear, nonlinear problems are essential to mathematicians, physicists, and engineers. However, solving nonlinear equations can be challenging and can result in fascinating events. In the analysis of complex nonlinear systems, the accurate solutions of the dynamic processes hold significant importance. Differential equations for linear and nonlinear systems can be solved using numerical, analytical, and semi-analytical methods. Numerous authors demonstrated different methods, including Laplace Adomian decomposition method, novel transform, Adomian decomposition method, Elzaki substitution method, homotopy perturbation Elzaki transform method, Homotopy Perturbation Method with Shehu Transform, Homotopy perturbation method,Homotopy analysis method, Sumudu decomposition method, Homotopy analysis transform method, Natural Method, differential transform method, Coupled Fractional Complex Homotopy perturbation method (FCT-HP), Fractional Residual Power Series Algorithm, Generalized differential transform method, Variational iteration method, Numerical method with NWS equation etc.

The Newell-Whitehead-Segel equation, also known as the NWS equation, is a partial differential equation that describes the behaviour of pattern formation in reaction-diffusion systems. It was proposed by scientists Robert Newell, John Whitehead, and Lee Segel in 1969. A Newell–Whitehead–Segal (NWS) equation comes under the form of nonlinear partial differentiation equation which mostly use in fluid mechanics.The equation can be written as:

$$\frac{\partial u}{\partial t} = D\nabla^2 u + f(u, v)$$

$$\frac{\partial v}{\partial t} = \gamma\nabla^2 v + g(u, v)$$

Here$u$ and v represent the concentrations of two interacting chemical substances or variables that diffuse through space. $\frac{\partial u}{\partial t}$ and $\frac{\partial v}{\partial t}$ are the temporal changes in u and v, respectively.

The terms D and γrepresent the diffusion coefficients of the substances. The Laplacian operator ∇^2 represents the spatial diffusion of the variables, which determines how they spread out or diffuse over time.The functions $f(u, v)$and $g(u, v)$represent the reaction terms that describe how the substances interact with each other. These functions usually involve non-linearities that can lead to complex pattern formation. The NWS equation is often used to model various biological and physical phenomena, such as the formation of animal coat patterns, the spatial distribution of chemical substances in biological systems, and the behaviour of certain physical systems exhibiting pattern formation.

Researchers have extensively studied the NWS equation and its solutions to gain insights into the mechanisms underlying pattern formation and self-organization in nature. The equation has been a valuable tool for understanding how complex patterns can emerge from simple local interactions and diffusion processes.

1. Preliminaries and Notations

The Elzaki transformation, also known as the Elzaki integral transform, is a mathematical technique used to solve ordinary form differential equations (ODEs) by transforming them into algebraic equations. It was introduced by the Sudanese mathematician Elzaki Ali Elzaki in the 1960s. The Elzaki transformation has been successfully applied to various fields of applied science and engineering, encompassing heat conduction, fluid dynamical mechanics, and electrical circuits. It provides an alternative approach to solving ODEs, particularly when analytical solutions are difficult to obtain using traditional methods. The use of the Elzaki transform in solving FPDEs can be particularly advantageous when analytical solutions are difficult to obtain using other methods. It provides a framework to transform the problem into an algebraic form, which can be tackled using well-established algebraic techniquesin this section, presenting some basic definition of Elzaki transform.

Definition: Elzaki Transformation

The Elzaki Transformation is precisely delineated concerning functions exhibiting exponential traits within the predefined set A [1]:

$$A = \left\{ f(t) : \exists\, M, k_1, k_2 > 0, |f(t)| < Me^{\frac{|t|}{k_j}}, \quad if\ t \in (-1)^j\, X\, [0, \infty) \right\}$$

The Elzaki Transformdefined as operator $E(g(\tau))$is,

$$E(g(\tau)) = v\int_0^{\infty} g(\tau)^{-\frac{\tau}{v}}\, d\tau = F(v) \quad ,\tau > 0$$

Exploring the Elzaki Transformation some function:

$f(t)$	$E(f(t))$
1	v^2
t	v^3
t^n	$n!\ v^{n+2}$

Property: Caputo Fractional Elzaki Transformation

Introducing the Elzaki Transformation of Caputo Fractional Derivative:

$$E\left[\frac{\partial^{\alpha}}{\partial\tau^{\alpha}} g(\tau)\right] = \frac{E\,[g\,(\tau)]}{v^{\alpha}} - \sum_{k=0}^{n-1} v^{k-\alpha+2}\ g^{(k)}(0), \quad n-1 < \alpha \le n \quad (1)$$

1.1. Fundamental Structure of the Modified Adomian Decomposition Elzaki Transformation Algorithm (MADETM) for Solving Nonlinear Fractional Differential Equations:

Let us consider a generic fractional non-linear partial differential equation as presented below:

$$D_t^{\alpha} w(x,t) + R[w(x,t)] + N[w(x,t)] = g(x,t) \quad (2)$$

With initial condition $w(x,0) = f(x)$(3)

Where $D_t^{\alpha} w(x,t)$ is Caputo fractional derivative of the function $w(x,t)$ defined as:

$$D_t^{\alpha} w(x,t) = \frac{\partial^{\alpha} w(x,t)}{\partial t^{\alpha}} = \begin{cases} \dfrac{1}{\Gamma(n-\alpha)}\displaystyle\int_{\alpha}^{t} (t-x)^{n-\alpha-1}\frac{\partial^{n} w(x,t)}{\partial t^{n}}dt & ,n-1 < \alpha < n \\ \dfrac{\partial^{n} w(x,t)}{\partial t^{n}} \alpha = n \in N \end{cases}$$

The source term is $g(x,t)$, the linear differential operator is represented by R, and the generic nonlinear differential operator is represented by N.

Taking the Elzaki Transform on both sides of Equation (2)

$$E\left[\frac{\partial^{\alpha} w(x,t)}{\partial t^{\alpha}}\right] + E\left[R[w(x,t)]\right] + E\left[N[w(x,t)]\right] = E[g(x,t)] \quad (4)$$

Using the properties of the Elzaki Transform on Equation (4)

$$E\ \frac{[w(x,t)]}{v^{\alpha}} - \sum_{k=0}^{n-1} v^{k-\alpha+2}\ w^{(k)}(x,0) = E[g(x,t)] - E\{[R[w(x,t)]] + [N[w(x,t)]]\}$$

$$E[w(x,t)] = \sum_{k=0}^{n-1} v^{k+2}\, w^{(k)}(x,0) + v^{\alpha} E[g(x,t)] - v^{\alpha} E\{[R[w(x,t)]] + [N[w(x,t)]]\} \quad (5)$$

Applying inverse Elzaki Transform on Equation (5)

$$E^{-1}[E\,(\mathrm{w}(\mathrm{x},\mathrm{t}))] = E^{-1}\left[\sum_{k=0}^{n-1} v^{k+2}\; w^{(k)}(x,0) + v^{\alpha}E[g(x,t)] - \; v^{\alpha}E\{[R[w(x,t)]] + [N[w(x,t)]]\}\right]$$

$$w(x,t) = \sum_{k=0}^{n-1}\frac{t^k}{k!}w^{(k)}(x,0)\; + E^{-1}(v^{\alpha}E\,[g(x,t)]) - E^{-1}(v^{\alpha}E\{[R[w(x,t)]] + [N[w(x,t)]]\}) \tag{6}$$

By applying MADM on equation the solution in infinite series given below

$$w(x,t) = \sum_{n=0}^{\infty} w_n(x,t) \tag{7}$$

$$N\,[w(x,t)] = \sum_{n=0}^{\infty} A_n \quad ;\text{ Where } A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}[N\sum_{i=0}^{\infty}\lambda^i u_i]\right]_{\lambda=0} \tag{8}$$

The nonlinear terms denoted by N are explained for employing the modified Adomian decomposition method for solving a nonlinear polynomial system following the utilization of the Elzaki transformation as specified below:

$$\{A_n\} = \{\,N_1(\,s_n) - \; N_1(\,s_{n-1})\} \tag{9}$$

Equation (6) is obtained by substituting Equations (7) and (8)

$$\sum_{n=0}^{\infty} w_n(x,t) = \; w(x,0) + E^{-1}(v^{\alpha}E[g(x,t)]) - E^{-1}\left(v^{\alpha}\;E\;\left\{R\left[\sum_{n=0}^{\infty} w_n(x,t)\right] + \left[\sum_{n=0}^{\infty} A_n\right]\right\}\right)$$

Analysing Both Perspectives of the Equation (9)

$$w_0(x,t) = w(x,0) + E^{-1}(v^{\alpha}E[g(x,t)])$$

$$w_1(x,t) = -E^{-1}(v^{\alpha}E\;\{R[w_0(x,t)] + A_0\})$$

$$w_2(x,t) = -E^{-1}(v^{\alpha}E\;\{R[w_1(x,t)] + A_1\})$$

$$w_n(x,t) = -E^{-1}(v^{\alpha}E\{R[w_{n-1}(x,t)] + A_{n-1}\}) \tag{10}$$

The analytic solution $w(x,t)$ is finally approximated using truncated series:

$$w(x,t) = \sum_{n=0}^{\infty} w_n(x,t) \tag{11}$$

1.2. Elzaki Transformation Method on NWS Equation

The operator form of the Newell-Whitehead-Segal (NWS) equation, based on the fractional model is define as:

$$w_t^{\alpha} = kw_{xx} + aw - bw^q, \quad t > 0, \;\; 0 < \alpha \le 1 \tag{12}$$

With initial Condition $w(x,0) = g(x)$ (13)

Where $w_t^{\alpha} = \frac{\partial^{\alpha} w}{\partial t^{\alpha}}$, The real numbers a, b, and k > 0 and the positive integer q are given.

By applying the Elzaki transform (ET) on equation (12),

$$E[w_t^{\alpha}] = E[kw_{xx} + aw - bw^q]$$

$$E\,\frac{[w(x,t)]}{v^{\alpha}} - \sum_{k=0}^{n-1} v^{k-\alpha+2}\;w^{(k)}(x,0) = E\{kw_{xx} + aw - bw^q\}$$

$E[w(x,t)] = \sum_{k=0}^{n-1} v^{k+2}\ w^{(k)}(x,0)\ + v^{\alpha}E\{kw_{xx} + aw - bw^{q}\}$ (14)

Applying inverse Elzaki Transform on Equation (14)

$$E^{-1}[E\ [w(x,t)]] = E^{-1}\left\{\sum_{k=0}^{n-1} v^{k+2}\ w^{(k)}(x,0)\ +\ v^{\alpha}E[kw_{xx} + aw - bw^{q}]\right\}$$

$w(x,t) = \sum_{k=0}^{n-1}\frac{t^k}{k!}w^{(k)}(x,0)\ + E^{-1}\{v^{\alpha}E\{kw_{xx} + aw - bw^{q}\}\}$ (15)

By applying MADM on equation the solution in infinite series given below

$w(x,t) = \sum_{n=0}^{\infty} w_n(x,t)$ (16)

The nonlinear term w^q is decomposed as:

$w^q(x,t) = \sum_{n=0}^{\infty} A_n$ (17)

The Adomian polynomials $A_n$is represented below.

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left[N\sum_{i=0}^{\infty}\lambda^i w_i\right]\right]_{\lambda=0}$$

Substituting (16) and (17) in equation (15)

$$\sum_{n=0}^{\infty} w_n(x,t) = \sum_{k=0}^{n-1}\frac{t^k}{k!}w^{(k)}(x,0)\ + E^{-1}\left(v^{\alpha}\ E\left[k\left(\sum_{n=0}^{\infty} A_n\right)_{xx} + a\left(\sum_{n=0}^{\infty} B_n\right) - b\left(\sum_{n=0}^{\infty} C_n\right)\right]\right) \quad (18)$$

Using (18), we define the following iterative formula.

$w_0(x,t) = E^{-1}[w(x,0)]$

$w_1(x,t) = E^{-1}\left[v^{\alpha}\ E\left[k\left(\sum_{n=0}^{\infty} A_0\right)_{xx} + a\left(\sum_{n=0}^{\infty} B_0\right) - b\left(\sum_{n=0}^{\infty} C_0\right)\right]\right]$

$$w_2(x,t) = E^{-1}\left[v^{\alpha}\ E\left[k\left(\sum_{n=0}^{\infty} A_1\right)_{xx} + a\left(\sum_{n=0}^{\infty} B_1\right) - b\left(\sum_{n=0}^{\infty} C_1\right)\right]\right]$$

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$w_n(x,t) = -E^{-1}\left[v^{\alpha}\ E\left[k\left(\sum_{n=0}^{\infty} A_{n-1}\right)_{xx} + a\left(\sum_{n=0}^{\infty} B_{n-1}\right) - b\left(\sum_{n=0}^{\infty} C_{n-1}\right)\right]\right]$(19)

After identifying these elements, replace them in $w(x,t) = \sum_{n=0}^{\infty} w_n(x,t)$to derive the solution in a sequential format.

2. Applications:

To illustrate the process of solution of the Modified Adomian Decomposition Elzaki Transformation Method (MADSTM), considered the system of nonlinear time fractional PDEs:

Example 3.1)We know that the Newell–Whitehead–Segal (NWS) equation,

$$\frac{\partial^{\alpha} w}{\partial t^{\alpha}} = k\frac{\partial^2 w}{\partial x^2} + aw - bw^q,\quad t > 0,\ \ 0 < \alpha \le 1$$

The above NWS equation can be transformed into the non-linear time-fractional Newell-Whitehead-Segel equation by changing the values of $b = 1, k = 2, h = -3, and\ a = 2.$

$$\frac{\partial^\alpha w}{\partial t^\alpha} = \frac{\partial^2 w}{\partial x^2} + 2w - 3w^2 \quad , t > 0, \ \ 0 < \alpha \le 1 \text{(20)}$$

With initial condition $u(x, 0) = \eta$ (21)

Solution: Using the process of Modified Adomian Decomposition Elzaki Transformation Method (MADETM) on Example 3.1, we get:

$$w_0(x, t) = \eta$$

$$w_1(x, t) = \frac{2\,\eta\, t^\alpha}{\lceil\alpha + 1} - \frac{3\,\eta^2\, t^\alpha}{\lceil\alpha + 1}$$

$$w_2(x, t) = \frac{4\,\eta\, t^{2\alpha}}{\lceil 2\alpha+1} - \frac{18\,\eta^2\, t^{2\alpha}}{\lceil 2\alpha+1} + \frac{18\,\eta^3\, t^{3\alpha}}{\lceil 3\alpha+1} \quad (22)$$

$$w_3(x, t) = \frac{8\,\eta\, t^{3\alpha}}{\lceil 3\alpha + 1} - \frac{60\,\eta^2\, t^{3\alpha}}{\lceil 3\alpha + 1} + \frac{144\,\eta^3\, t^{3\alpha}}{\lceil 3\alpha + 1} - \frac{108\,\ \eta^4\, t^{3\alpha}}{\lceil 4\alpha + 1} + \frac{12\,\eta^2\, t^{3\alpha}\lceil 2\alpha + 1}{\lceil(\alpha + 1)^2\lceil 3\alpha + 1} - \frac{36\,\eta^3\, t^{3\alpha}\lceil 2\alpha + 1}{\lceil(\alpha + 1)^2\lceil 3\alpha + 1} + \frac{27\,\eta^4\, t^{3\alpha}\lceil 2\alpha + 1}{\lceil(\alpha + 1)^2\lceil 3\alpha + 1}$$

$$w_4(x, t) = \frac{16\,\eta\, t^{4\alpha}}{\lceil 4\alpha + 1} - \frac{168\,\eta^2\, t^{4\alpha}}{\lceil 4\alpha + 1} + \frac{648\,\eta^3\, t^{4\alpha}}{\lceil 4\alpha + 1} - \frac{1080\,\eta^4\, t^{4\alpha}}{\lceil 4\alpha + 1} + \frac{648\,\eta^5\, t^{4\alpha}}{\lceil 4\alpha + 1} - \frac{24\,\eta^2\, t^{4\alpha}\lceil 2\alpha + 1}{\lceil(\alpha + 1)^2\lceil 4\alpha + 1} + \frac{144\,\eta^3\, t^{4\alpha}\lceil 2\alpha + 1}{\lceil(\alpha + 1)^2\lceil 4\alpha + 1} - \frac{270\,\eta^4\, t^{4\alpha}\lceil 2\alpha + 1}{\lceil(\alpha + 1)^2\lceil 4\alpha + 1} w(x, t)$$

$$= \eta + \frac{2\,\eta\, t^\alpha}{\lceil\alpha + 1} - \frac{3\,\eta^2\, t^\alpha}{\lceil\alpha + 1} + \frac{4\,\eta\, t^{2\alpha}}{\lceil 2\alpha + 1} - \frac{18\,\eta^2\, t^{2\alpha}}{\lceil 2\alpha + 1} + \frac{18\,\eta^3\, t^{3\alpha}}{\lceil 3\alpha + 1} + \frac{8\,\eta\, t^{3\alpha}}{\lceil 3\alpha + 1} - \frac{60\,\eta^2\, t^{3\alpha}}{\lceil 3\alpha + 1} + \frac{144\,\eta^3\, t^{3\alpha}}{\lceil 3\alpha + 1} - \frac{108\,\ \eta^4\, t^{3\alpha}}{\lceil 4\alpha + 1} + \frac{12\,\eta^2\, t^{3\alpha}\lceil 2\alpha + 1}{\lceil(\alpha + 1)^2\lceil 3\alpha + 1} - \frac{36\,\eta^3\, t^{3\alpha}\lceil 2\alpha + 1}{\lceil(\alpha + 1)^2\lceil 3\alpha + 1} + \frac{27\,\eta^4\, t^{3\alpha}\lceil 2\alpha + 1}{\lceil(\alpha + 1)^2\lceil 3\alpha + 1} + \frac{16\,\eta\, t^{4\alpha}}{\lceil 4\alpha + 1} - \frac{168\,\eta^2\, t^{4\alpha}}{\lceil 4\alpha + 1} + \frac{648\,\eta^3\, t^{4\alpha}}{\lceil 4\alpha + 1} - \frac{1080\,\eta^4\, t^{4\alpha}}{\lceil 4\alpha + 1} + \frac{648\,\eta^5\, t^{4\alpha}}{\lceil 4\alpha + 1} - \frac{24\,\eta^2\, t^{4\alpha}\lceil 2\alpha + 1}{\lceil(\alpha + 1)^2\lceil 4\alpha + 1} + \frac{144\,\eta^3\, t^{4\alpha}\lceil 2\alpha + 1}{\lceil(\alpha + 1)^2\lceil 4\alpha + 1} - \frac{270\,\eta^4\, t^{4\alpha}\lceil 2\alpha + 1}{\lceil(\alpha + 1)^2\lceil 4\alpha + 1} + \frac{162\,\eta^5\, t^{4\alpha}\lceil 2\alpha + 1}{\lceil(\alpha + 1)^2\lceil 4\alpha + 1} - \frac{48\,\eta^2\, t^{4\alpha}\lceil 3\alpha + 1}{\lceil\alpha + 1\lceil 2\alpha + 1\lceil 4\alpha + 1} + \frac{288\,\eta^3\, t^{4\alpha}\lceil 3\alpha + 1}{\lceil\alpha + 1\lceil 2\alpha + 1\lceil 4\alpha + 1} - \frac{540\,\eta^4\, t^{4\alpha}\lceil 3\alpha + 1}{\lceil\alpha + 1\lceil 2\alpha + 1\lceil 4\alpha + 1} + \frac{324\,\eta^5\, t^{4\alpha}\lceil 3\alpha + 1}{\lceil\alpha + 1\lceil 2\alpha + 1\lceil 4\alpha + 1} - \cdots + \frac{162\,\eta^5\, t^{4\alpha}\lceil 2\alpha + 1}{\lceil(\alpha + 1)^2\lceil 4\alpha + 1} - \frac{48\,\eta^2\, t^{4\alpha}\lceil 3\alpha + 1}{\lceil\alpha + 1\lceil 2\alpha + 1\lceil 4\alpha + 1} + \frac{288\,\eta^3\, t^{4\alpha}\lceil 3\alpha + 1}{\lceil\alpha + 1\lceil 2\alpha + 1\lceil 4\alpha + 1} - \frac{540\,\eta^4\, t^{4\alpha}\lceil 3\alpha + 1}{\lceil\alpha + 1\lceil 2\alpha + 1\lceil 4\alpha + 1} + \frac{324\,\eta^5\, t^{4\alpha}\lceil 3\alpha + 1}{\lceil\alpha + 1\lceil 2\alpha + 1\lceil 4\alpha + 1}$$

Therefore,Series representation of the solution $w\ (x, t)$ is as follows:

$$w(x, t) = \ w_0(x, t) + w_1(x, t) + w_2(x, t) + \ w_3(x, t) + \ w_4(x, t) + \cdots \ldots \ldots \text{(23)}$$

In particular when$\alpha = 1$, we get the solution in the form:

$$w(x, t) = \ \eta + \frac{2\eta t}{\lceil 2} - \frac{3\,\eta^2 t}{\lceil 2} + \frac{4\,\eta\, t^2}{\lceil 3} - \frac{18\,\eta^2\, t^2}{\lceil 3} + \frac{18\,\eta^3\, t^3}{\lceil 4} + \frac{8\,\eta\, t^3}{\lceil 4} - \frac{60\,\eta^2\, t^3}{\lceil 4} + \frac{144\,\eta^3\, t^3}{\lceil 4} - \frac{108\,\ \eta^4\, t^3}{\lceil 5} + \frac{12\,\eta^2\, t^3\lceil 3}{\lceil(2)^2\lceil 4} - \frac{36\,\eta^3\, t^3\lceil 3}{\lceil(2)^2\lceil 4} + \frac{27\,\eta^4\, t^3\lceil 3}{\lceil(2)^2\lceil 4} + \frac{16\,\eta\, t^4}{\lceil 5} - \frac{168\,\eta^2\, t^4}{\lceil 5} + \frac{648\,\eta^3\, t^4}{\lceil 5} - \frac{1080\,\eta^4\, t^4}{\lceil 5} + \frac{648\,\eta^5\, t^4}{\lceil 5} - \frac{24\,\eta^2\, t^4\lceil 3}{\lceil(2)^2\lceil 5} +$$

$$\frac{144\,\eta^3\,t^4\Gamma3}{\Gamma(2)^2\Gamma5} - \frac{270\,\eta^4\,t^4\Gamma3}{\Gamma(2)^2\Gamma5} + \frac{162\,\eta^5\,t^4\Gamma3}{\Gamma(2)^2\Gamma5} - \frac{48\,\eta^2\,t^4\Gamma4}{\Gamma2\Gamma3\Gamma5} + \frac{288\,\eta^3 t^4\Gamma4}{\Gamma2\Gamma3\Gamma5} - \frac{540\,\eta^4 t^4\Gamma4}{\Gamma2\Gamma3\Gamma5} + \frac{324\,\eta^5 t^4\Gamma4}{\Gamma2\Gamma3\Gamma5} - \cdots. \quad (24)$$

The convergent solution to the classical Newell-Whitehead-Segel equation is achieved rapidly for the given equation(20)

$$w(x,t) = \frac{-\frac{2}{3}\eta t^{2t}}{-\frac{2}{3}+\eta-\eta t^{2t}} \quad (25)$$

Example 3.2) Taking $a = 0, b > 0$ and $q = 3$, in Equation (12) becomes,

$$\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial x^2} - bw^3 \quad (26)$$

With initial condition $\mathrm{w(x, 0)} = \sqrt{\frac{2}{b}}\left(\frac{2x}{x^2+1}\right)$

Solution: Applying the previously mentioned method to Example 2

$$w_0(x,t) = \sqrt{\frac{2}{b}}\left(\frac{2x}{x^2+1}\right)$$

$$w_1(x,t) = -\sqrt{\frac{2}{b}}\frac{12xt}{(x^2+1)^2}$$

$$w_2(x,t) = \sqrt{\frac{2}{b}}\frac{864\,x^3t^4}{(x^2+1)^6} - \sqrt{\frac{2}{b}}\frac{576\,x^3t^3}{(x^2+1)^5} + \sqrt{\frac{2}{b}}\frac{72\,\,xt^2}{(x^2+1)^3} \quad (27)$$

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Therefore, the solution w (x, t)in series form is given by.

$$w(x,t) = w_0(x,t) + w_1(x,t) + w_2(x,t) + w_3(x,t) + \cdots \ldots \ldots$$

$$w\,(x,t) = \sqrt{\frac{2}{b}}\left(\frac{2x}{x^2+1}\right) - \sqrt{\frac{2}{b}}\frac{12xt}{(x^2+1)^2} + \sqrt{\frac{2}{b}}\frac{864\,x^3t^4}{(x^2+1)^6} - \sqrt{\frac{2}{b}}\frac{576\,x^3t^3}{(x^2+1)^5} + \sqrt{\frac{2}{b}}\frac{72\,\,x\,t^2}{(x^2+1)^3} - \cdots. \quad (28)$$

The closed form solution for Equation (28) is precisely determined.

$$w\,(x,t) = \sqrt{\frac{2}{b}}\left(\frac{2x}{x^2+6t+1}\right) \quad (29)$$

3. Result and Discussion

In Table 1, result comparison for Exact solution, MADETM and Fractional Complex Transform- He's polynomials method (FCT-HPM) and its relative absolute error shown at $\eta = 0.001$ $and \alpha = 1$. In Table 2, result comparison for Exact solution, MADETM and Natural Homotopy Perturbation Method (NHPM) and its relative absolute error shown at $\eta = 0.001$ $and \alpha = 1$. In Table 3, result comparison for Exact solution, MADETM and VIM and its relative absolute error shown at $a = 0, b > 0$ as x increases up to 1 and t increases from 0 to 1.

Table 1. On comparing the results between Exact solution, MADETM and FCT-HPM [27] for$\eta = 0.001\ and\ \alpha = 1$ and for Example 3.1

x	t	Exact Solution	Approx. Solution by MADETM	Re. Absolute Error	Approx. Solution by FCT- HPM [27]	Re. Absolute Error
0.2	0.16	0.00137635	0.00137632	2.17968E-05	0.00137396	0.001736477
	0.32	0.00189393	0.00189299	0.000496322	0.00187473	0.01013765
	0.48	0.0026054	0.0025978	0.002917019	0.00253977	0.02518999
	0.64	0.00358269	0.00354881	0.009456582	0.0034232	0.04451683
	0.8	0.00492384	0.00481432	0.022242802	0.004599	0.065972899
	0.96	0.00676192	0.00677288	0.001620841	0.0061642	0.088395012

Table 2. Comparison of results between Exact solution, MADETM and NHPTM [25] at $\eta = 0.001\ and\ \alpha = 1$ and for Example 3.1

x	T	Exact Solution	Approx. Solution by MADETM	Re. Absolute Error	Approx. Solution by NTHPM [25]	Re. Absolute Error
0.2	0.16	0.00137635	0.00137632	2.17968E-05	0.001454302	0.056636902
	0.32	0.00189393	0.00189299	0.000496322	0.002307619	0.218428928
	0.48	0.0026054	0.0025978	0.002917019	0.003755086	0.441270285
	0.64	0.00358269	0.00354881	0.009456582	0.005991837	0.672440848
	0.8	0.00492384	0.00481432	0.022242802	0.009213009	0.871102351
	0.96	0.00676192	0.00677288	0.001620841	0.013613735	1.013294301

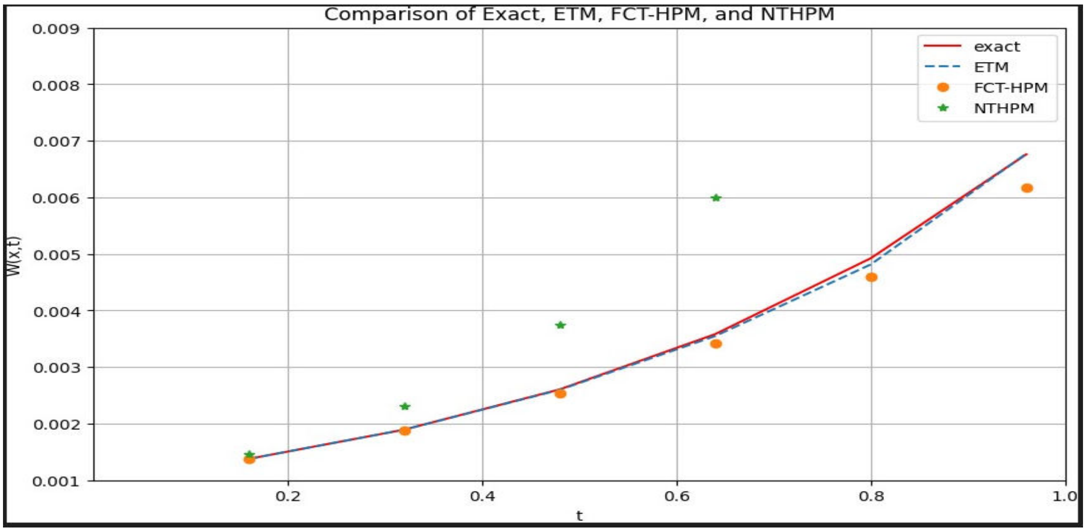

Figure1. Graph plot of Exact, MADETM and FCT-HPM and NHPTM at $\eta = 0.001$ and $\alpha = 1$ or Example 3.1

Table 3. The outcomes comparison between MADETM and VTM [28] for Example 3.2

x	Exact	Approx. Solution by MADETM	Re. Absolute Error	Approx. Solution by VTM	Re. Absolute Error
2	1.0101525	1.0099788	0.000171954	1.0098764	0.000273325
4	0.64282435	0.64281907	8.21375E-06	0.64281695	1.15117E-05
6	0.45134475	0.45134479	8.86241E-08	0.45134464	2.43716E-07
8	0.34493014	0.34493044	8.69741E-07	0.34493041	7.82767E-07
10	0.2783885	0.27838877	9.69868E-07	0.27838877	9.69868E-07

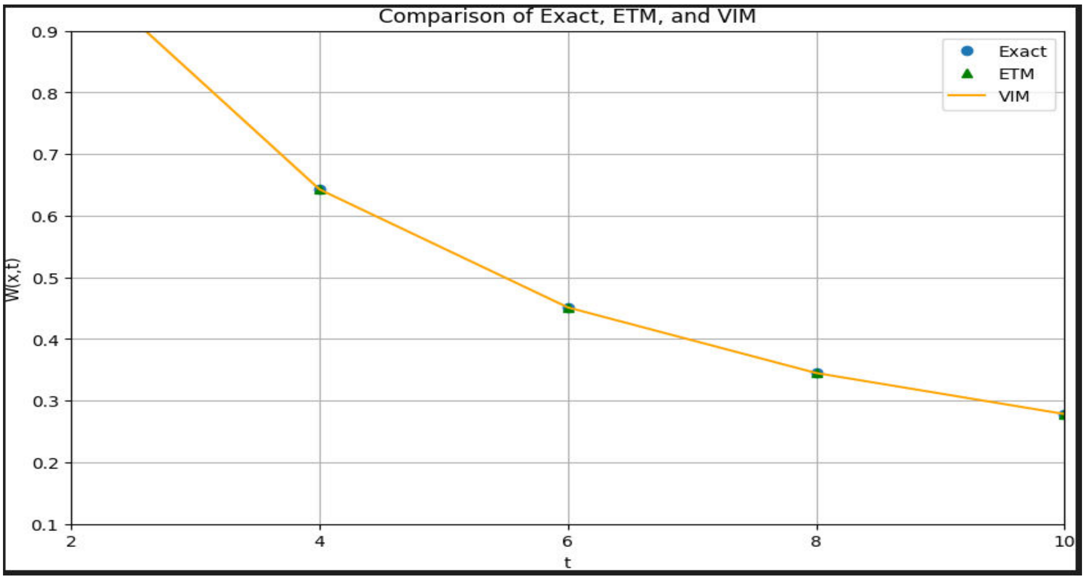

Figure 2. Comparison between Exact, MADETM and VTM for Example 3.2

4. Analysis and Conclusion:

The below table presented values of exact and approximate solutions for adopted technique to displays the results of calculated performance metrics for different values of t. Based on the provided data, there seems to in the relationship between the dependent variable t and the independent variable$x$. The descriptive statistics and correlation analysis suggest that $'t'$ varies systematically with changes in $'x'$.

Table 4. MADETM and Exact solutions for example 3.1 with 20 number of observations:

Samples	x	t	exact	MADETM	Re. Absolute Error
1	0.2	0.16	0.00137635	0.00137632	2.17968E-05
2	0.2	0.32	0.00189393	0.00189299	0.000496322
3	0.2	0.48	0.0026054	0.0025978	0.002917019
4	0.2	0.64	0.00358269	0.00354881	0.009456582
5	0.2	0.8	0.00492384	0.00481432	0.022242802
6	0.2	0.96	0.00676192	0.00677288	-0.001620841
7	0.2	1.12	0.007223509	0.0071703	7.36604E-03
8	0.2	1.28	0.008280505	0.008218809	0.007450799
9	0.2	1.44	0.009337501	0.009267317	0.007516368
10	0.2	1.6	0.010394498	0.010315826	0.007568601
11	0.2	1.76	0.011451494	0.011364334	0.007611193
12	0.2	1.92	0.01250849	0.012412843	0.007646585
13	0.2	2.08	0.013565486	0.013461351	7.67646E-03
14	0.2	2.24	0.014622483	0.01450986	0.007702021
15	0.2	2.4	0.015679479	0.015558369	0.007724133
16	0.2	2.56	0.016736475	0.016606877	0.007743452
17	0.2	2.72	0.017793472	0.017655386	0.007760476
18	0.2	2.88	0.018850468	0.018703894	0.007775591
19	0.2	3.04	0.019907464	0.019752403	7.78910E-03
20	0.2	3.2	0.02096446	0.020800911	0.007801248

Additionally, the hypothesis testing provides insights into the nature of this relationship.

H_0 : The solutions exhibit no significant difference.

H_A : A significant difference exists between the solutions.

Based on the above assumption for the statistical observations are as follows:

The p-value, being less than 0.0001, signifies an exceedingly strong level of statistical significance according to conventional criteria. Its Confidence Interval: The mean of MADETM with Exact solution = 0.00008291570, 95% confidence interval difference: 0.00005735939 to 0.00010847201

T=6.7907, Degree of freedom =19, Standard error of difference=0.000

Group	MADETM	Exact Equation
Mean	0.0109229970	0.01084008000
SD	0.00625751807	0.00620776354
SEM	0.00139922375	0.00138809813
N	20	20

Table 5. MADETM and Exact solutions for example 3.2 with 20 number of observations:

sample	x	Exact	Approximate Solution by ETM	Re. Absolute Error	Approximate Solution by VIM
1	2	1.0101525	1.0099788	0.000171954	1.0098764
2	4	0.64282435	0.64281907	8.21375E-06	0.64281695
3	6	0.45134475	0.45134479	8.86241E-08	0.45134464
4	8	0.34493014	0.34493044	8.69741E-07	0.34493041
5	10	0.2783885	0.27838877	9.69868E-07	0.27838877
6	12	0.017101385	0.017171767	-0.004115573	0.017212894
7	14	-0.159040836	-0.158935102	0.000664823	-0.158873286
8	16	-0.335183057	-0.335041971	-0.000420922	-0.334959466
9	18	-0.511325278	-0.51114884	-0.00034506	-0.511045646
10	20	-0.687467499	-0.687255709	-0.000308073	-0.687131826
11	22	-0.86360972	-0.863362578	0.000286173	-0.863218006
12	24	-1.039751941	-1.039469447	0.000271694	-1.039304186
13	26	-1.215894162	-1.215576316	-0.000261409	-1.215390366
14	28	-1.392036383	-1.391683185	-0.000253728	-1.391476546
15	30	-1.568178604	-1.567790054	-0.000247772	-1.567562726
16	32	-1.744320825	-1.743896923	0.000243018	-1.743648906
17	34	-1.920463046	-1.920003792	0.000239137	-1.919735086
18	36	-2.096605267	-2.096110661	-0.000235908	-2.095821266
19	38	-2.272747488	-2.27221753	-0.000233179	-2.271907446
20	40	-2.448889709	-2.448324399	-0.000230843	-2.447993626

The P value is less than 0.0001. By conventional criteria, this difference is extremely statistically significant. The Confidence Interval: The mean of MADETM with Exact solution = 0.00022946600, 95% confidence interval difference: 0.00013091751 to 0.00032801449
Indeterminate values: T= 4.87535, Degree of freedom =19, Standard error of difference=0.000

Group	MADETM	Exact Equation
Mean	-0.77530914350	-0.77553860950
SD	1.04275900519	1.04296893345
SEM	0.23316800198	0.23321494336
N	20	20

In this study, successfully applied semi analytical technique on fractional order NWS equationusing modified Adomain Decomposition Elzaki Transformation method. In the calculation part up to 4th iterations approximate solution are carried out. Approximate solutions are rapidly converging to exact solutions for then said problems. For the example 3.1, obtained results are compared with FCT-HP method and NHPM method to identify the accuracy of results. In example 3.2, results are compared with VIM method to find out the accuracy. Relative absolute error is also measured for NWS equation at various conditions. The graph is plotted to analyse the results obtained. To validate the result for selected applications employed an independent samples t-test to assess the significance of the observed differences between Exact solution and Approximate solution. The t-test was chosen due to its appropriateness for comparing means between two independent groups, aligning with our research design. The t-test results revealed a significant disparity in the mean scores between the two factors, implying a meaningful divergence in the outcomes linked to the two conditions.

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