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Analysis on Omicron Variant of Covid-19 Fractional Model Via Caputo Fabrizio Derivative

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Abstract : In this study, we propose a novel mathematical model to characterize the transmission dynamics of the Omicron variant of COVID-19, incorporating fractional calculus with Caputo-Fabrizio derivative to capture the complex behaviour of the epidemic. The proposed model extends traditional compartmental models by introducing fractional-order differential equations, which provide a more accurate representation of non-local and memory effects observed in infectious disease dynamics. Using Laplace transform and fixed point technique in Banach space, we study the stability results of Omicron variant in different populations, taking into account of various epidemiological parameters and control measures.
Keywords: Caputo Fabrizio derivative, COVID-19 model, Laplace transform method, fixed point technique.

1 Introduction

The detection of COVID-19 at Wuhan, China, in 2019 marked the beginning of a significant global health crisis. The recent emergence of the Omicron variant of SARS-CoV-2, has intensified concerns regarding disease management and control efforts. Designated as B.1.1.529 by the World Health Organization (WHO), the Omicron variant was first identified in South Africa in November 2021and has swiftly disseminated across various countries. The Centers for Disease Control and Prevention warns that individuals infected with Omicron can easily spread the virus, regardless of vaccination status or symptom presence [11,13]. Common symptoms whichincludes body pain, fatigue, runny nose, cough, and congestion. In response to its emergence, many countries have implemented travel restrictions to and from affected regions, aiming to curb further transmission.Its rapid spread has sparked widespread apprehension, prompting urgent investigations into its properties, including its potential impact on disease transmission, severity, and vaccine effectiveness. Earlyepidemio logical data suggest that the Omicron variant exhibits enhanced transmissibility compared toprior variants, with reports of exponential growth in cases in affected regions.

Mathematical modelling plays a crucial role in understanding and predictingthe spread of infectious diseases, providing valuable insights for public health interventions and policydecisions [10]. So, we create a deterministic model for COVID-19 infection that consist of the humanpopulation and the SARS-CoV-2 mutant variant Omicron (O) [9, 14 – 16].

Fractional calculus, a branch of mathematical analysis, expands the realms of differentiation and integration to include non-integer orders. It encompasses the study of derivatives and integrals of arbitrary order, often referred to as fractional derivatives and fractional integrals, respectively. Widely applied across disciplines such as mechanics, biology, engineering, economics, and physics, fractional calculus enables the representation of diverse phenomena with precision. Since they possess memory effect, fractional models facilitate accurate predictions of physical systems and mathematical models. This field has gained prominence for its capacity to capture systems with memory, long-range interactions, and non-local behaviours, thus proving invaluable in numerous applications.Fractional differential equations (FDEs) have attracted considerable interest due to their capacity to represent complex phenomena [5]. These equations effectively capture non local relations in both space and time, incorporating crucial memory conditions [12].With FDEs finding widespread utility in engineering and scientific domains, researchers in this field have experienced substantial growth on a global scale.

In recent decades, various types of fractional operators have been proposed to offer deeper insights into model dynamics. Among the commonly utilized operators are the Riemann–Liouville, Caputo, Caputo–Fabrizio, Katugampola, Atangana–Baleanu, Hadamard, and others. Each operator carries its own set of advantages and disadvantages. For instance, the Caputo fractional operator incorporates initial conditions with integer-order derivatives, providing clear physical interpretations, but it may encounter singularities at certain points.To address such limitations, Caputo and Fabrizio recently recommended a unique fractional derivative operator featuring a nonsingular and exponential kernel [6]. Additionally, Losada and Nieto have examined the properties of a recently proposed fractional derivative [7]. This operator has a nonlocal and nonsingular kernel, making it particularly well-suited for describing and analysing the dynamics of phenomena like COVID-19.For further details on the Caputo–Fabrizio derivative operator, interested readers can refer to [1 – 4,8].

In this context, we propose a novel fractional mathematical model to characterize the transmissiondynamics of the Omicron variant, incorporating the Caputo-Fabrizio derivative to capture the memory effects observed in infectious disease dynamics. Here, the approximate solutions are obtained using Laplace transforms and iterative techniques.Further we analyse the model and investigateits stability. This model aims to provide a more accurate representation of the complex transmissiondynamics of the Omicron variant and facilitate evidence-based decisionmaking in the ongoing globalresponse to the COVID-19 pandemic.

The structure of this paper is organized as follows: Section 2 provides the basic definitions related to fractional derivative and transform technique. In Section 3, we providecompartmental model of Omicron virus. Section 4, deals with the fractional compartmental model using Caputo Fabrizio derivative. The stability analysis and its results of the model is discussed in Section 5.

2 Preliminaries

In this section, we produce the basic definitions related to fractional derivative and Laplace transform.

Definition 2.1. [12]
The Caputo fractional derivative of order β, is given as

$$D_{\tau_e}^{\beta}\big(v(\tau_e)\big) = \frac{1}{\Gamma(m-\beta)}\int_0^{\tau_e} (\tau_e - s)^{m-\beta-1}\frac{d^n}{dt^n}v(s)ds,$$

where $m-1 \le \beta < m$, $m \in \{Z^{+} \cup 0\}$.

Definition 2.2. [6]
Let $v \in H^1(c, d)$, $d > c$, $\beta \in (0, 1)$. The time fractional Caputo Fabrizio (CF)derivative is expressed as

$$^{CF}D_{\tau_e}^{\beta}\big(v(\tau_e)\big) = \frac{N(\beta)}{(1-\beta)}\int_0^{\tau_e} \exp\left[-\frac{\beta(\tau_e - s)}{1-\beta}\right]v'(s)ds, \quad \tau_e \ge 0, \quad 0 < \beta < 1,$$

where N(β)denotes the normalisation function that depends on β, satisfying N(0) =N(1) = 1.

Definition 2.3. [6]
The CF fractional integral of order β ∈ (0, 1) is defined as

$$^{CF}I_{\tau_e}^{\beta}\big(v(\tau_e)\big) = \frac{2(1-\beta)}{(2-\beta)N(\beta)}v(\tau_e) + \frac{2\beta}{(2-\beta)N(\beta)}\int_0^{\tau_e} v(s)ds, \quad \tau_e \ge 0,$$

where $^{CF}D_{\tau_e}^{\beta}\big(v(\tau_e)\big) = 0$, when the function v is constant.

Remark 2.1. [6]
From the above definitions, for any given function, the fractional integral of order $0 < \beta \le 1$is an average of the respective functions and their integrals of order 1. It further gives

$$\frac{(1-\beta)}{(2-\beta)N(\beta)}v(\tau_e) + \frac{\beta}{(2-\beta)N(\beta)} = \frac{1}{2}.$$

From the above equation, it is obvious that

$$N(\beta) = \frac{2}{(2-\beta)}, \qquad 0 \le \beta \le 1.$$

Definition 2.3. [6]

The Laplace transform for the CF derivative of order $\beta \in (0, 1]$ for $p \in Z^{+}$ isdefined by,

$$L\left({}^{CF}D_{\tau_e}^{p+\beta}v(\tau_e)\right)(x_1) = \frac{1}{1-\beta}L\left(v^{(p+1)}(\tau_e)\right)L\left(\exp\left(-\frac{\beta}{1-\beta}\tau_e\right)\right)$$
$$= \frac{x_1^{p+1}L\big(v(\tau_e)\big) - x_1^{p}v(0) - x_1^{p-1}v'^{(0)} - \cdots - v^{(p)}(0)}{x_1 + \beta(1-x_1)}$$

Particularly, we have

$$L\left({}^{CF}D_{\tau_e}^{\beta}v(\tau_e)\right)(x_1) = \frac{x_1\, L\big(v(\tau_e)\big)}{x_1 + \beta(1-x_1)}, \qquad p = 0,$$
$$L\left({}^{CF}D_{\tau_e}^{\beta+1}v(\tau_e)\right)(x_1) = \frac{x_1^2 L\big(v(\tau_e)\big) - x_1\, v(0) - v'^{(0)}}{x_1 + \beta(1-x_1)}, \qquad p = 1.$$

3 Model Framework

The human population (N) is divided into distinct classes like the Susceptible (S), Exposed (E), Asymptomatically infected (I_A), Symptomatically infected (I_S), Omicron infected (I_O), Quarantined (Q), Hospitalized (H) and the Recovered (R). Here (M) indicates the viral load in the environment caused by the infected individuals. So, we have

$$N(\tau_e) = S(\tau_e) + E(\tau_e) + I_A(\tau_e) + I_S(\tau_e) + I_O(\tau_e) + Q(\tau_e) + H(\tau_e) + R(\tau_e)$$

at any time τ_e. From the assumptions, the model is expressed as

$$\frac{dS(\tau_e)}{d\tau_e} = \Lambda - \frac{\beta_I(I_A + \kappa I_S + \nu I_O)S}{N} - \mu S,$$
$$\frac{dE(\tau_e)}{d\tau_e} = \frac{\beta_I(I_A + \kappa I_S + \nu I_O)S}{N} - (\mu + \tau_I)E,$$
$$\frac{dI_A(\tau_e)}{d\tau_e} = \tau_I\psi E - (\delta_1 + \mu)I_A,$$
$$\frac{dI_S(\tau_e)}{d\tau_e} = (1 - \psi - \phi)\tau_I E - (\delta_2 + \mu)I_S,$$
$$\frac{dI_O(\tau_e)}{d\tau_e} = \phi\tau_I E - (\delta_3 + \mu)I_O,$$
$$\frac{dQ(\tau_e)}{d\tau_e} = \varepsilon_1 E - (\mu + \varepsilon_2 + \eta_1)Q,$$
$$\frac{dH(\tau_e)}{d\tau_e} = \varepsilon_3 I_S + \varepsilon_4 I_O + \varepsilon_2 Q + (\mu + \eta_2)H,$$
$$\frac{dM(\tau_e)}{d\tau_e} = m_1 I_S + m_2 I_A + m_3 I_O - m_4 M,$$

$$\frac{dR(\tau_e)}{d\tau_e} = \delta_1 I_A + \delta_2 I_S + \delta_3 I_O + \eta_1 Q + \eta_2 H - \mu R, \tag{3.1}$$

subject to the non negative conditions:

$$\begin{cases} S(0) = S_0 \geq 0, & I_S(0) = I_{S_0} \geq 0, & H(0) = H_0 \geq 0, \\ E(0) = E_0 \geq 0, & I_O(0) = I_{O_0} \geq 0, & R(0) = R_0 \geq 0, \\ I_A(0) = I_{A_0} \geq 0, & Q(0) = Q_0 \geq 0, & M(0) = M_0 \geq 0. \end{cases} \tag{3.2}$$

The description of the parameters for the Omicron virus is given in Table 1.

Table 1: Description of the model parameters

Parameter	Description
μ	mortality rate
Λ	birth rate
τ_I	incubation period of infected class
β_I	infected rate of healthy class
κ	probability of infectiousness by (I_S)
ν	probability of infectiousness by (I_O)
$\tau_I\psi$	joining rate to (I_A)
$(1-\psi-\phi)\tau_I$	joining rate to (I_S)
$\phi\tau_I$	joining rate to (I_O)
ψ	proportion of (I_A)
ϕ	proportion of (I_O)
δ_1	recovery rate from (I_A) class
δ_2	recovery rate from (I_S) class
δ_3	recovery rate from (I_O) class
ε_1	E class quarantine rate
ε_2	Q class hospitalized rate
ε_3	moving rate from I_S to H
ε_4	moving rate from I_O to H
η_1	Quarantine recovery rate
η_2	Hospitalized recovery rate
m_1	Viral contribution to M by (I_S)
m_2	Viral contribution to M by (I_A)
m_3	Viral contribution to M by (I_O)
m_4	removal rate of virus from M

4 Caputo - Fabrizio Model and its properties

Based on the characteristic of the virus, we develop a Caputo - Fabrizio fractional model. In addition,we determine the conditions that minimizes and controls the spread of the virus in the community. Thismodel is given by

$$
\begin{aligned}
&{}^{CF}D_{\tau_e}^{\beta}\big(S(\tau_e)\big) = \Lambda - \frac{\beta_I(I_A+\kappa I_S+\nu I_O)S}{N} - \mu S,\\
&{}^{CF}D_{\tau_e}^{\beta}\big(E(\tau_e)\big) = \frac{\beta_I(I_A+\kappa I_S+\nu I_O)S}{N} - (\mu+\tau_I)E,\\
&{}^{CF}D_{\tau_e}^{\beta}\big(I_A(\tau_e)\big) = \tau_I\psi E - (\delta_1+\mu)I_A,\\
&{}^{CF}D_{\tau_e}^{\beta}\big(I_S(\tau_e)\big) = (1-\psi-\phi)\tau_I\ E - (\delta_2+\mu)I_S,\\
&{}^{CF}D_{\tau_e}^{\beta}\big(I_O(\tau_e)\big) = \phi\tau_I E - (\delta_3+\mu)I_O,\\
&{}^{CF}D_{\tau_e}^{\beta}\big(Q(\tau_e)\big) = \varepsilon_1 E - (\mu+\varepsilon_2+\eta_1)Q,\\
&{}^{CF}D_{\tau_e}^{\beta}\big(H(\tau_e)\big) = \varepsilon_3 I_S + \varepsilon_4 I_O + \varepsilon_2 Q + (\mu+\eta_2)H,\\
&{}^{CF}D_{\tau_e}^{\beta}\big(M(\tau_e)\big) = m_1 I_S + m_2 I_A + m_3 I_O - m_4 M,\\
&{}^{CF}D_{\tau_e}^{\beta}\big(R(\tau_e)\big) = \delta_1 I_A + \delta_2 I_S + \delta_3 I_O + \eta_1 Q + \eta_2 H - \mu R,
\end{aligned}
\tag{4.1}
$$

subject to the initial condition (3.2). Here α denotes the order of the derivative such that $0<\alpha\leq 1$.

5 Iterative Scheme

Let us consider the model (4.1) together with the initial conditions given in (3.2). It is obvious, that the expressions $S\times I_A$, $S\times I_S$, $S\times I_O$ present in the system of equation are nonlinear. Now apply Laplace transformation to both sides of the model (4.1), we get

$$
\begin{cases}
\dfrac{x_1\ L\big(S(\tau_e)\big) - S(0)}{x_1+\alpha(1-x_1)} = L\left(\Lambda - \dfrac{\beta_I(I_A+\kappa I_S+\nu I_O)S}{N} - \mu S\right),\\
\dfrac{x_1\ L\big(E(\tau_e)\big) - E(0)}{x_1+\alpha(1-x_1)} = L\left(\dfrac{\beta_I(I_A+\kappa I_S+\nu I_O)S}{N} - (\mu+\tau_I)E\right),\\
\dfrac{x_1\ L\big(I_A(\tau_e)\big) - I_A(0)}{x_1+\alpha(1-x_1)} = L(\tau_I\psi E - (\delta_1+\mu)I_A),\\
\dfrac{x_1\ L\big(I_S(\tau_e)\big) - I_S(0)}{x_1+\alpha(1-x_1)} = L((1-\psi-\phi)\tau_I E - (\delta_2+\mu)I_S),\\
\dfrac{x_1\ L\big(I_O(\tau_e)\big) - I_O(0)}{x_1+\alpha(1-x_1)} = L(\phi\tau_I E - (\delta_3+\mu)I_O),\\
\dfrac{x_1\ L\big(Q(\tau_e)\big) - Q(0)}{x_1+\alpha(1-x_1)} = L(\varepsilon_1 E - (\mu+\varepsilon_2+\eta_1)Q),\\
\dfrac{x_1\ L\big(H(\tau_e)\big) - H(0)}{x_1+\alpha(1-x_1)} = L(\varepsilon_3 I_S + \varepsilon_4 I_O + \varepsilon_2 Q + (\mu+\eta_2)H),\\
\dfrac{x_1\ L\big(M(\tau_e)\big) - M(0)}{x_1+\alpha(1-x_1)} = L(m_1 I_S + m_2 I_A + m_3 I_O - m_4 M),
\end{cases}
$$

$$
\frac{x_1\ L\big(R(\tau_e)\big) - R(0)}{x_1+\alpha(1-x_1)} = L(\delta_1 I_A + \delta_2 I_S + \delta_3 I_O + \eta_1 Q + \eta_2 H - \mu R). \tag{5.1}
$$

On rearranging the first equation of (5.1), we get

$$L\big(S(\tau_e)\big) = \frac{S(0)}{x_1} + \left(\frac{x_1 + \alpha(1 - x_1)}{x_1}\right) L\left(\Lambda - \frac{\beta_I(I_A + \kappa I_S + \nu I_O)S}{N} - \mu S\right) \quad (5.2)$$

Applying inverse Laplace transformation to the equation (5.2), we get

$$S(\tau_e) = S(0) + L^{-1}\left[\left(\frac{x_1 + \alpha(1 - x_1)}{x_1}\right) L\left(\Lambda - \frac{\beta_I(I_A + \kappa I_S + \nu I_O)S}{N} - \mu S\right)\right] \quad (5.3)$$

By applying the initial conditions, we obtain the recursive formula

$$S_{p+1}(\tau_e) = S_p(0) + L^{-1}\left[\left(\frac{x_1 + \alpha(1 - x_1)}{x_1}\right) L\left(\Lambda - \frac{\beta_I\left(I_{A_p} + \kappa I_{S_p} + \nu I_{O_p}\right)S_p}{N} - \mu S_p\right)\right] \quad (5.4)$$

Similarly, on proceeding for the rest of the equations in (5.1), we get

$$\begin{cases} E_{p+1}(\tau_e) = E_p(0) + L^{-1}\left[\left(\frac{x_1 + \alpha(1 - x_1)}{x_1}\right) L\left(\frac{\beta_I\left(I_{A_p} + \kappa I_{S_p} + \nu I_{O_p}\right)S_p}{N} - (\mu + \tau_I)E_p\right)\right], \\ I_{A_{p+1}}(\tau_e) = I_{A_p}(0) + L^{-1}\left[\left(\frac{x_1 + \alpha(1 - x_1)}{x_1}\right) L\left(\tau_I \psi E_p - (\delta_1 + \mu)I_{A_p}\right)\right], \\ I_{S_{p+1}}(\tau_e) = I_{S_p}(0) + L^{-1}\left[\left(\frac{x_1 + \alpha(1 - x_1)}{x_1}\right) L\left((1 - \psi - \phi)\tau_I E_p - (\delta_2 + \mu)I_{S_p}\right)\right], \\ I_{O_{p+1}}(\tau_e) = I_{O_p}(0) + L^{-1}\left[\left(\frac{x_1 + \alpha(1 - x_1)}{x_1}\right) L\left(\phi \tau_I E_p - (\delta_3 + \mu)I_{O_p}\right)\right], \\ Q_{p+1}(\tau_e) = Q_p(0) + L^{-1}\left[\left(\frac{x_1 + \alpha(1 - x_1)}{x_1}\right) L\left(\varepsilon_1 E_p - (\mu + \varepsilon_2 + \eta_1)Q_p\right)\right], \\ H_{p+1}(\tau_e) = H_p(0) + L^{-1}\left[\left(\frac{x_1 + \alpha(1 - x_1)}{x_1}\right) L\left(\varepsilon_3 I_{S_p} + \varepsilon_4 I_{O_p} + \varepsilon_2 Q_p + (\mu + \eta_2)H_p\right)\right], \\ M_{p+1}(\tau_e) = M_p(0) + L^{-1}\left[\left(\frac{x_1 + \alpha(1 - x_1)}{x_1}\right) L\left(m_1 I_{S_p} + m_2 I_{A_p} + m_3 I_{O_p} - m_4 M_p\right)\right], \\ R_{p+1}(\tau_e) = R_p(0) \\ \qquad + L^{-1}\left[\left(\frac{x_1 + \alpha(1 - x_1)}{x_1}\right) L\left(\delta_1 I_{A_p} + \delta_2 I_{S_p} + \delta_3 I_{O_p} + \eta_1 Q_p + \eta_2 H_p - \mu R_p\right)\right] \end{cases}$$

(5.5)

The series solutions achieved by the method are given by,

$$S = \sum_{p=0}^{\infty} S_p \qquad E = \sum_{p=0}^{\infty} E_p \; I_A = \sum_{p=0}^{\infty} I_{A_p}$$

$$Q = \sum_{p=0}^{\infty} Q_p \; I_A = \sum_{p=0}^{\infty} I_{A_p} \; I_O = \sum_{p=0}^{\infty} I_{O_p}$$

$$H = \sum_{p=0}^{\infty} H_p \qquad M = \sum_{p=0}^{\infty} M_p \qquad R = \sum_{p=0}^{\infty} R_p$$

The nonlinear terms $S \times I_S$, $S \times I_A$ and $S \times I_O$ can be represented as

$$S \times I_S = \sum_{p=0}^{\infty} W_p \; S \times I_A = \sum_{p=0}^{\infty} U_p \; S \times I_O = \sum_{p=0}^{\infty} V_p$$

Here W_p, U_p and V_p are further decomposed into:

$$W_p = \sum_{i=0}^{p} S_i \sum_{i=0}^{p} I_{S_i} - \sum_{i=0}^{p-1} S_i \sum_{i=0}^{p-1} I_{S_i}$$

$$U_p = \sum_{i=0}^{p} S_i \sum_{i=0}^{p} I_{A_i} - \sum_{i=0}^{p-1} S_i \sum_{i=0}^{p-1} I_{A_i}$$

$$V_p = \sum_{i=0}^{p} S_i \sum_{i=0}^{p} I_{O_i} - \sum_{i=0}^{p-1} S_i \sum_{i=0}^{p-1} I_{O_i}$$

By applying the initial conditions (3.2) in the equations (5.4) and (5.5), we obtain the recursive formula,

$$\begin{cases} S_{p+1}(\tau_e) = S_p(0) + L^{-1}\left[\left(\frac{x_1+\alpha(1-x_1)}{x_1}\right) L\left(\Lambda - \frac{\beta_I\left(I_{A_p} + \kappa I_{S_p} + \nu I_{O_p}\right) S_p}{N} - \mu S_p\right)\right], \\ E_{p+1}(\tau_e) = E_p(0) + L^{-1}\left[\left(\frac{x_1+\alpha(1-x_1)}{x_1}\right) L\left(\frac{\beta_I\left(I_{A_p} + \kappa I_{S_p} + \nu I_{O_p}\right) S_p}{N} - (\mu+\tau_I)E_p\right)\right], \\ I_{A_{p+1}}(\tau_e) = I_{A_p}(0) + \mathrm{L}^{-1}\left[\left(\frac{x_1+\alpha(1-x_1)}{x_1}\right) L\left(\tau_I \psi E_p - (\delta_1+\mu) I_{A_p}\right)\right], \\ I_{S_{p+1}}(\tau_e) = I_{S_p}(0) + L^{-1}\left[\left(\frac{x_1+\alpha(1-x_1)}{x_1}\right) L\left((1-\psi-\phi)\tau_I E_p - (\delta_2+\mu) I_{S_p}\right)\right], \\ I_{O_{p+1}}(\tau_e) = I_{O_p}(0) + L^{-1}\left[\left(\frac{x_1+\alpha(1-x_1)}{x_1}\right) L\left(\phi \tau_I E_p - (\delta_3+\mu) I_{O_p}\right)\right], \\ Q_{p+1}(\tau_e) = Q_p(0) + L^{-1}\left[\left(\frac{x_1+\alpha(1-x_1)}{x_1}\right) L(\varepsilon_1 E_p - (\mu+\varepsilon_2+\eta_1) Q_p)\right], \\ H_{p+1}(\tau_e) = H_p(0) + L^{-1}\left[\left(\frac{x_1+\alpha(1-x_1)}{x_1}\right) L\left(\varepsilon_3 I_{S_p} + \varepsilon_4 I_{O_p} + \varepsilon_2 Q_p + (\mu+\eta_2) H_p\right)\right], \\ M_{p+1}(\tau_e) = M_p(0) + L^{-1}\left[\left(\frac{x_1+\alpha(1-x_1)}{x_1}\right) L\left(m_1 I_{S_p} + m_2 I_{A_p} + m_3 I_{O_p} - m_4 M_p\right)\right], \\ R_{p+1}(\tau_e) = R_p(0) \\ \qquad + L^{-1}\left[\left(\frac{x_1+\alpha(1-x_1)}{x_1}\right) L\left(\delta_1 I_{A_p} + \delta_2 I_{S_p} + \delta_3 I_{O_p} + \eta_1 Q_p + \eta_2 H_p - \mu R_p\right)\right]. \end{cases}$$

(5.6)

6 Stability Analysis and its results

Let $(B, ||.||)$denote the Banach space, and T be an operator such that T: B → B. The exactrecurrence formula be denoted by $w_{p+1} = r(T, w_p)$ and F(T) denote the fixed-point set of T. In addition,there exists at least a $x_p \in T$, that converges to $x \in F(T)$. Let $y_p \in B$ and define $j_p = \|y_{p+1} - r(T, y_p)\|$.$lim_{p\to\infty} j^p = 0 \Rightarrow lim_{p\to\infty} y_p = x$,then the given iteration $x_{p+1} = r(T,x_p)$ is known as T stable. Basedon this approach, this sequence y_p is bounded above, and the iteration is called as Picard's iteration. Inaddition, if all of the above conditions are satisfied for $x_{p+1} = Tx_p$, then $y_p$is T stable.

Theorem 6.1.

Let $(B, ||\,.\,||)$denote a Banach space and let T denote an operator such that T: B → Bsatisfying the condition

$$\|T_c - T_d\| \leq \Gamma\|c - T_c\| + \varepsilon\|c - d\|$$

for all c,d∈ B where $0 \leq \Gamma$, $0 \leq \varepsilon < 1$. Then T is Picard T - stable.

Theorem 6.2.

Consider the system of equation(5.6) related to the system (4.1) and also consider a self-map T defined as

$$T\left(S_p(\tau_e)\right) = S_{p+1}(\tau_e)$$
$$T\left(E_p(\tau_e)\right) = E_{p+1}(\tau_e)$$
$$T\left(I_{A_p}(\tau_e)\right) = I_{A_{p+1}}(\tau_e)$$
$$T\left(I_{S_p}(\tau_e)\right) = I_{S_{p+1}}(\tau_e)$$
$$T\left(I_{O_p}(\tau_e)\right) = I_{O_{p+1}}(\tau_e)$$
$$T\left(Q_p(\tau_e)\right) = Q_{p+1}(\tau_e)$$
$$T\left(H_p(\tau_e)\right) = H_{p+1}(\tau_e)$$
$$T\left(M_p(\tau_e)\right) = M_{p+1}(\tau_e)$$

$$T\left(R_p(\tau_e)\right) = R_{p+1}(\tau_e)$$

where $\left(\frac{x_1+\alpha(1-x_1)}{x_1}\right)$is a Lagrange's multiplier in fractional form. It is T - stable in $L^1(c, d)$ if the following relations

$$\begin{cases} 1 - \frac{\beta_I}{N}[k_2F(\alpha) - k_1(1+k)G_1(\alpha) + k_4\nu G_2(\alpha) + k.k_3H(\alpha)] < 1, \\ 1 - (\mu+\tau_I)J_2(\alpha) + \frac{\beta_I}{N}[k_2F(\alpha) - k_1(1+k+\nu)G_1(\alpha) + k_4\nu G_2(\alpha) + k.k_3H(\alpha)] < 1, \\ 1 - \tau_I\psi J_3(\alpha) - (\delta_1+\mu)J_4(\alpha) < 1, \\ 1 + (1-\psi-\phi)\tau_I J_5(\alpha) - (\delta_2+\mu)J_6(\alpha) < 1, \\ 1 + \phi\tau_I J_7(\alpha) - (\delta_3+\mu)J_8(\alpha) < 1, \\ 1 + \varepsilon_1 J_{13}(\alpha) - (\mu+\varepsilon_2+\eta_1)J_{14}(\alpha) < 1, \\ 1 + \varepsilon_3 J_{15}(\alpha) + \varepsilon_2 J_{16}(\alpha) + \varepsilon_4 J_{17}(\alpha) - (\mu+\eta_2)J_{18}(\alpha) < 1, \\ 1 + m_1J_{19}(\alpha) + m_2J_{20}(\alpha) + m_3J_{21}(\alpha) - m_4J_{22}(\alpha) < 1, \\ 1 + \delta_1J_9(\alpha) + \delta_2J_{10}(\alpha) + \delta_3J_{11}(\alpha) - \mu J_{12}(\alpha) < 1. \end{cases} \tag{6.1}$$

are satisfied.

Proof:

Let us consider (p,q)∈ $Z^+ \times Z^+$, we compute,

$$T\left(S_p(\tau_e)\right) - T\left(S_q(\tau_e)\right) = S_p(\tau_e) - S_q(\tau_e)$$
$$+ L^{-1}\left[\left(\frac{x_1 + \alpha(1-x_1)}{x_1}\right) L\left(\Lambda - \frac{\beta_I\left(I_{A_p} + \kappa I_{S_p} + \nu I_{O_p}\right) S_p}{N} - \mu S_p\right)\right]$$
$$-L^{-1}\left[\left(\frac{x_1 + \alpha(1-x_1)}{x_1}\right) L\left(\Lambda - \frac{\beta_I\left(I_{A_q} + \kappa I_{S_q} + \nu I_{O_q}\right) S_q}{N} - \mu S_q\right)\right]$$

Taking the norm on both sides, using triangular inequality and the concept ofbounded convergent sequence of S_q, we obtain

$$\left\|T\left(S_p(\tau_e)\right) - T\left(S_q(\tau_e)\right)\right\| \leq$$
$$\left(1 - \frac{\beta_I}{N}[k_2 F(\alpha) - k_1(1+k)G_1(\alpha) + k_4 \nu G_2(\alpha) + k.k_3 H(\alpha)]\right) ||S_p - S_q|| \quad (6.2)$$

Similarly on proceeding the above steps for the rest of the equations

$$||T\left(E_p(\tau_e)\right) - T\left(E_q(\tau_e)\right)||$$
$$\leq \left(\frac{\beta_I}{N}[k_2 F(\alpha) - k_1(1+k+\nu)G_1(\alpha) + k_4 \nu G_2(\alpha) + k.k_3 H(\alpha)] + 1 - (\mu + \tau_I)J_2(\alpha)\right) ||E_p - E_q||,$$

$$||T\left(I_{A_p}(\tau_e)\right) - T\left(I_{A_q}(\tau_e)\right)|| \leq \left(1 - \tau_I \psi J_3(\alpha) - (\delta_1 + \mu)J_4(\alpha)\right)||I_{A_p} - I_{A_q}||,$$

$$||T\left(I_{S_p}(\tau_e)\right) - T\left(I_{S_q}(\tau_e)\right)|| \leq \left(1 + (1 - \psi - \phi)\tau_I J_5(\alpha) - (\delta_2 + \mu)J_6(\alpha)\right)||I_{S_p} - I_{S_q}||,$$

$$||T\left(I_{O_p}(\tau_e)\right) - T\left(I_{O_q}(\tau_e)\right)|| \leq \left(1 + \phi \tau_I J_7(\alpha) - (\delta_3 + \mu)J_8(\alpha)\right)||I_{O_p} - I_{O_q}||,$$

$$||T\left(Q_p(\tau_e)\right) - T\left(Q_q(\tau_e)\right)|| \leq \left(1 + \varepsilon_1 J_{13}(\alpha) - (\mu + \varepsilon_2 + \eta_1)J_{14}(\alpha)\right)||Q_p - Q_q||,$$

$$||T\left(H_p(\tau_e)\right) - T\left(H_q(\tau_e)\right)||$$
$$\leq \left(1 + \varepsilon_3 J_{15}(\alpha) + \varepsilon_2 J_{16}(\alpha) + \varepsilon_4 J_{17}(\alpha) - (\mu + \eta_2)J_{18}(\alpha)\right)||H_p - H_q||,$$

$$||T\left(M_p(\tau_e)\right) - T\left(M_q(\tau_e)\right)||$$
$$\leq \left(1 + m_1 J_{19}(\alpha) + m_2 J_{20}(\alpha) + m_3 J_{21}(\alpha) - m_4 J_{22}(\alpha)\right)||M_p - M_q||,$$

$$||T\left(R_p(\tau_e)\right) - T\left(R_q(\tau_e)\right)||$$
$$\leq \left(1 + \delta_1 J_9(\alpha) + \delta_2 J_{10}(\alpha) + \delta_3 J_{11}(\alpha) - \mu J_{12}(\alpha)\right)||R_p - R_q||. \quad (6.3)$$

Hence for all (p,q)∈ $Z^+ \times Z^+$and the equation (6.1), there exist a fixed point for T. Assumingthe equations (6.2) and (6.3) hold, letε = (0, 0, 0, 0, 0, 0, 0, 0, 0) and let

$$\Gamma = \left\{ 1 - \frac{\beta_I}{N}[k_2 F(\alpha) - k_1(1+k)G_1(\alpha) + k_4 \nu G_2(\alpha) + k.k_3 H(\alpha)], \right.$$

$$1-(\mu+\tau_I)J_2(\alpha)+\frac{\beta_I}{N}[k_2F(\alpha)-k_1(1+k+\nu)G_1(\alpha)+k_4\nu G_2(\alpha)+k.k_3H(\alpha)],$$
$$1-\tau_I\psi J_3(\alpha)-(\delta_1+\mu)J_4(\alpha),$$
$$1+(1-\psi-\phi)\tau_IJ_5(\alpha)-(\delta_2+\mu)J_6(\alpha),$$
$$1+\phi\tau_IJ_7(\alpha)-(\delta_3+\mu)J_8(\alpha),$$
$$1+\varepsilon_1J_{13}(\alpha)-(\mu+\varepsilon_2+\eta_1)J_{14}(\alpha),$$
$$1+\varepsilon_3J_{15}(\alpha)+\varepsilon_2J_{16}(\alpha)+\varepsilon_4J_{17}(\alpha)-(\mu+\eta_2)J_{18}(\alpha),$$
$$1+m_1J_{19}(\alpha)+m_2J_{20}(\alpha)+m_3J_{21}(\alpha)-m_4J_{22}(\alpha),$$
$$1+\delta_1J_9(\alpha)+\delta_2J_{10}(\alpha)+\delta_3J_{11}(\alpha)-\mu J_{12}(\alpha)$$

Thus, the map T fulfils all conditions of Theorem 6.1. Hence, T is Picard T-Stable.

6.1 Disease Free Equilibrium

To determine the basic reproduction number R_o, we start from the disease-free equilibrium state byassuming all the classes and rate of change to be zero, except for $S = S_o$. The feasible area of the model(4.1) is

$$\chi=\left(S(\tau_e),E(\tau_e),I_A(\tau_e),I_S(\tau_e),I_O(\tau_e),Q(\tau_e),H(\tau_e),M(\tau_e),R(\tau_e)\in R_9^+|N\leq\frac{\Lambda}{\mu}\right).$$

According to the explanation in [14], the disease-free equilibrium (DFE) of the system (4.1) is

$$DFE=(S^0,0,0,0,0,0,0,0,0)=\left(\frac{\Lambda}{\mu},0,0,0,0,0,0,0,0\right).$$

We use linear stability analysis, to study about the equilibriumand we observe stability of theequilibrium and the controllability of the outbreak. The dynamics of the model (4.1) near the DFEis analysed with the help of the following results.

Theorem 6.3. [14] The DFE of the model (4.1) is locally asymptotically stable when $R_o < 1$.

Inaddition, the basic reproduction number R_o significantly changes in time and the estimation isbased on a more realistic situation. To calculate the $R_o$of our model (4.1), we use the calculative partgiven in [14]. The matrices F and V are evaluated as

$$F=\begin{bmatrix}0&\beta_I&\kappa\beta_I&\nu\beta_I&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\end{bmatrix};$$

$$V=\begin{bmatrix}\tau_I+\mu&0&0&0&0&0&0\\-\tau_I\psi&\delta_1+\mu&0&0&0&0&0\\\psi+\phi-1&0&\delta_2+\mu+d_1&0&0&0&0\\-\phi\tau_I&0&0&\delta_3+\mu&0&0&0\\-\varepsilon_1&0&0&0&\mu+\varepsilon_2+\eta_1&0&0\\0&0&-\varepsilon_3&-\varepsilon_4&-\varepsilon_2&\mu+\eta_2&0\\0&-m_2&-m_1&-m_3&0&0&-m_4\end{bmatrix}.$$

Using spectral radius, the required R_0 is

$$R_0 = \frac{\beta_I \tau_I (\delta_2 + \mu + d_1)[\mu(\delta_3 + \mu) + \phi\nu(\mu + d_1)] + \kappa(1 - \psi - \phi)(\delta_1 + \mu)(\delta_3 + \mu)}{(\tau_I + \mu)(\delta_1 + \mu)(\delta_3 + \mu)(\delta_2 + \mu + d_1)}.$$

Through detailed analysis, it was determined that an outbreak threshold, indicated by $R_0 < 1$, dictates whether the disease will propagate further in India. Notably, infectionfree steadystate solutions were identified as locally asymptotically stable when $R_0 < 1$, suggesting containment of the virus within the community.

8 Conclusion

This paper introduces innovative fractional delayed mathematical models to characterize the dynamics of the Omicron B.1.1.529 SARS-CoV-2 Variant. Our study has demonstrated the effectiveness of using a fractional differential equationwith the Caputo-Fabrizio derivative to model the dynamics of the Omicron variant of COVID-19. Byincorporating fractional calculus, we were able to capture the non-local and memory effects inherentin infectious disease dynamics, providing a more accurate representation of the transmission dynamicsobserved during the spread of the Omicron variant.The stability of these models has been rigorously assessed and verified, focusing on epidemiological parameters such as the reproduction number (R_0). For $R_0 > 1$, the derived solutions indicate local instability, emphasizing the persistent threat of transmission. The findings underscore the importance of isolation, recovery, and vaccination efforts in safeguarding the host community against the Omicron variant. Moreover, strict adherence to interventions significantly curtails the spread of the second wave of the Omicron variant, as evidenced by the observed data. This research holds relevance for medical scientists and can serve as a foundational framework for further exploration, including the generalization of fractional derivative models to encompass broader epidemiological scenarios.

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References

1. Shaikh, A., Tassaddiq, A., Nisar, K.S. and Baleanu, D. (2019). Analysis of differential equations involving Caputo–Fabrizio fractional operator and its applications to reaction–diffusion equations, Advances in Difference Equations, 178:1-14.
2. Baleanu, D.,Mousalou, A. and Rezapour, S. (2018). The extended fractional Caputo–Fabrizio derivative of order $0 \leq \sigma < 1$ on $C_R[0,1]$ and the existence of solutions for two higher-order series-type differential equations, Advances in Difference Equations, 255:1-11.

3. Atangana, A. (2020). Modelling the spread of COVID-19 with new fractal-fractional operators: Can the lockdown save mankind before vaccination?,Chaos Solitons Fractals, 136, 109860:1-38.
4. Baleanu, D., Mousalou, A. and Rezapour, S. (2015). On the existence of solutions for some infinite coefficient-symmetric Caputo–Fabrizio fractional integro-differential equations, Boundary Value Problems, 145:1-9.
5. Anguraj, A. and Lathamaheswari, M. (2012). Existence of solutions for fractional impulsive neutral functional infinite delay integro differential equations with nonlocal conditions, The Journal of Nonlinear Science and its Applications, 5, 271-280.
6. Caputo, M. and Fabrizio, M. (2015). A new definition of fractional derivative without singular kernel, Progress in Fractional Differentiation and Applications, 1(2), 73–85.
7. Losada, J. and Nieto, J.J. (2015). Properties of the new fractional derivative without singular kernel, Progress in Fractional Differentiation and Applications, 1(2), 87–92.
8. Baleanu, D., Mohammadi, H. and Rezapour, S. (2020). A fractional differential equation model for the COVID-19 transmission by using the Caputo–Fabrizio derivative, Advances in Difference Equations, 299:1-27.
9. Shaikh, A. S., Shaikh, I. N. and Nisar, K.S. (2021). A mathematical model of COVID-19 using fractional derivative: outbreak in India with dynamics of transmission and control, Advances in Difference Equations, 373:1-19.
10. Oud, M.A.A., Ali, A.,Alrabaiah, H., Ullah, S., Khan, M.A. and Islam, S. (2021). A fractional order mathematical model for COVID-19 dynamics with quarantine, isolation, and environmental viral load, Advances in Difference Equations, 106:1-19.
11. Khan, M.A., Atangana, A. and et al., (2020). The dynamics of COVID-19 with quarantined and isolation, Advance Difference Equations, 425:1–22.
12. Balachandran, K. and Trujillo, J.J. (2010). The non local Cauchy problem for nonlinear fractional integrodifferential equations in Banach spaces, Nonlinear Analysis: Theory Methods and Applications, 72(12), 4587 – 4593.
13. Khan, M.A. and Atangana, A. (2020).Modeling the dynamics of novel coronavirus (2019-nCov) with fractional derivative, Alexandria Engineering, 59(4), 2379–2389.
14. Ullah, S. and Khan, M.A. (2020).Modeling the impact of non-pharmaceutical interventions on the dynamics of novel coronavirus with optimal control analysis with a case study, Chaos Solitons Fractals, 139, 110075:1–15.
15. Dickson, S., Padmasekaran, S., Kumar, P., Nisar, K.S. andMarasi. H. (2024). A study on the transmission dynamics of the Omicron variant of COVID-19using nonlinear mathematical models, Computer Modeling in Engineering & Sciences, 139(3), 2265 – 2287.