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Numerical Result of Boundary Layer Stagnation Point Flow and Heat Transfer Over an Exponentially Stretching Sheet With Combined Effect of Magnetic Field and Thermal Radiation

Ambuja Joshi*

Department of Mathematics, Shankar Narayan College of Arts and Commerce, Bhayandar West Thane Maharashtra INDIA

Abstract
Numerical solution of MHD momentum and thermal boundary layer flow and heat transfer over an exponentially stretching sheet, with combined effect of Magnetic Field and Thermal Radiation are considered for investigation. The influence of various flow and heat transfer parameters are analysed with the usage of graphs.
Key Words: Exponentially stretching sheet; heat source/sinkparameter; Runge-Kutta shooting method; Prandtl Number; Magnetic Field Parameter.

I Introduction

The study of flow over a stretching sheet has generated numerous interest in recent years in view of its numerous industrial applications such as the aerodynamic extrusion of plastic sheets,the boundary layer ,condensation process of metallic plate in cooling bath and also in polymer industries. The boundary layer flow over a stretching sheet was studied by various researchers as mentioned in from [1] to [10], The studies of thermal radiation and heat transfer plays an important role in electrical power generation,astrophysical flows ,solar power technology and other industries areas.

In this paper,we investigate numerically the combined effect of magnetic field and thermal radiation on two dimensional boundary layer flow and heat transfer over an exponentially stretching sheet. By applying similarity transformation,the boundary layer equations, which are PDE's are converted into ODE's and are solved by numerically using Runge-Kutta method

2. Mathematical formulation and solution

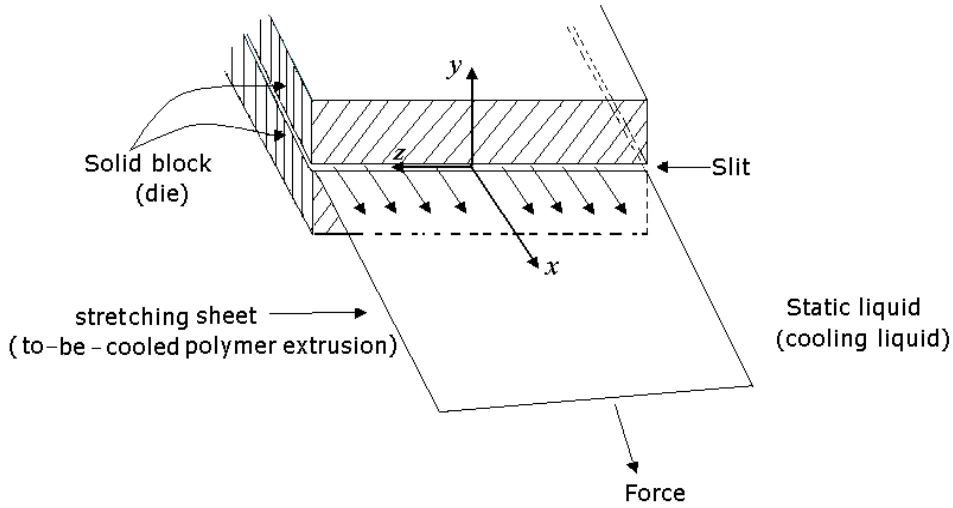

Fig 1. Schematic diagram of stretching sheet

Let us consider the laminar flow of viscous incompressible fluid past a flat and impressible elastic sheet. By applying two equal and opposite forces along the x-axis the sheet is stretched with a speed $u_w(x)$ proportional to the distance from the origin x=0. The resulting motion of the otherwise quiescent fluid is caused by the moving sheet, and the flow is governed by steady two-dimensional flow. The viscous fluid is only partially adhering to the stretching sheet. Under the usual boundary layer approximations ,the flow and heat transfer with radiation effects are governed by following equations,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=u_e\frac{du_e}{dx}+\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-(u_e-u) \tag{2.2}$$

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{2.3}$$

Where u and v are the velocities in the x and y –direction,respectively , ρ is the fluid density, υ is the kinametic viscocity , μ is the dynamic viscocity, T is the temperature, k is the thermal conductivity, C_p is specific heat and q_r is the radiative heat flux.The boundary condition given by,

$$\left.\begin{aligned} &u=u_w(x), v=0, T=T_w(x), at\ y=0 \\ &u\to u_e(x), T\to T_\infty as\ \mathrm{y}\to\infty \end{aligned}\right\} \tag{2.4}$$

$$u_w(x)=U_0 e^{\frac{x}{L}}, u_e(x)=U_s e^{\frac{x}{L}}, T_w(x)=T_\infty+(T_0+T_\infty)e^{\frac{2x}{L}} \quad (2.5)$$

$$q_r=-\frac{4\sigma *}{3k *}\frac{\partial T^4}{\partial y} \quad where \quad T^4 \cong 4T_\infty^3-3T_\infty^4$$

$$q_r=-\frac{4\sigma *}{3k *}4T_\infty^3\frac{\partial T^4}{\partial y}=-\frac{16\sigma * T_\infty^3}{3k *}\frac{\partial T}{\partial y} \quad (2.6)$$

Where U_0 ,T_0 and L are the reference velocity,temperature and length respectively.The radiative heat flux q_r is simplified by Rossnald approximation.

$$q_r=-\frac{4\sigma^*}{3k^*}.4T_\infty^3.\frac{\partial T^4}{\partial y} \quad (2.7)$$

The approximation is valid at points optically far from the boundary surface and it is good for intensive absorption,which is for an optically thick boundary layer .It is assumed that the temperature difference with in ye flow such that the term T^4 may be expressed as linear fuction of temperature.Hence,expanding T^4 by Taylor series about T_∞ and neglecting higher order terms gives

$$T^4 \cong 4T_\infty^3-3T_\infty^4 \quad (2.8)$$

The equation (1) is the continuity equation is identically satisfied if we chose the stream fuction ψ

Such that,

$$u=\frac{\partial \psi}{\partial y} \text{ and } v=-\frac{\partial \psi}{\partial x} \quad (2.9)$$

The momentum and energy equations can be transformed into the corresponding ordinary differential equations by including the following similarity equations,

$$\eta=\sqrt{\frac{\mathrm{Re}\, y}{2L}}e^{\frac{x}{2L}}, \psi(x,\eta)=\sqrt{2\mathrm{Re}\, v}e^{\frac{x}{2L}}f(\eta), \quad (2.10)$$

$$T(x,y)=T_\infty+(T_0-T_\infty)e^{\frac{ax}{2L}}\theta(\eta), \quad (2.11)$$

The equations (2.1)-(2.5) and are transformed into ordinary differential equation with the aid of equations (2.9)-(2.11). Thus the governing equations are,

$$f'''+ff''-2f'^2+Gr\theta+2A^2=0 \quad (.2.12)$$

$$\theta'' - \frac{\Pr}{1+Nr}[Ecf''^2 - 4f'\theta + f\theta] = 0 \quad (2.13)$$

The boundary conditions (2.4) reduces to,

$$f(0)=0, f'(0)=1, \theta(0)=1, \quad (2.14)$$

$$f'(\infty)=0, \theta(\infty)=0, \quad (2.15)$$

Where prime(') denote the differentiation with respect to η and dimensionless parameters are:

$Gr = \frac{2g\beta(T_0 - T_\infty)L}{U_0^\alpha}$ is the Grashof number , $M = \frac{\sigma B_0^2 L}{\rho U_0}$ and $\Pr = \frac{\mu c_p}{k}$ is the Prandtlnumber, $A = \frac{U_s}{U_0}$ $\mathrm{Re} = \frac{U_0 L}{\upsilon}$ is the Reynolds number.

3. Numerical Solution

In this study,an efficient Runge-Kutta fifth order method along with shooting technique has been employed to analyze the flow of model for the above coupled ordinary differential equations (2.12) & (2.13) for the different values of governing parameters viz.Prandtl number Pr, Etc.

4.4. Results and Discussions

Fig.(2). and Fig.(3).It is observed that increase in Grashof number enhances velocity and temperature, in both momentum and thermal boundary layers respectively. Additionally, it is pragmatic that this increase in velocity is because of velocity difference between stretched sheet and the adjoining fluid. Grashof number leads to boost in velocity and further it leads to increase in thickness of boundary layer

Fig. (4) and Fig. (5) Represents the influence of Prandtl number *Pr* on transfer of heat in thermal boundary layer and is evident from these plots, that large values of Prandtl number consequences in decrease in temperature and Velocity of the flow field. Since it is allready known fact that, the thermal boundary layer thickness is inversely proportional to the square root of Prandtl number, The decrease of temperature profile with Pr is clear-cut in both cases.

Fig.(6). Shows the influence of parameter Nr on velocity profile, and it is observed from this figure that, the velocity is an increasing function of parameter Nr, which offers confrontation to the flow ensuing in decrease of velocity in momentum boundary layer,which concurs with the results of various authors.

Fig.(7). Shows the influence of Nr in momentum boundary layer. From this figure it is observed that, the effect of Nr is to enhance the temperature profile in thermal boundary layer.

Fig.(8).and Fig.(9). Explores the influence of Eckert number Ec on velocity and temperature profiles respectively. Enhancement of values of Ec, results in enhancement of both momentum and thermal boundary layer thicknesses respectively. In fig 8, Ec converges at 0.2.

Fig.(10). explores the variation of temperature profile with Prandtl number Pr.This graph de¦picts that Prandtl number is a decreasing function of temperature.This is because of the fact that a fluids with higher Prandtl number has comparatively low thermal conductivity ,which reduces conduction and there by the thermal boundary layer thickness decreases.

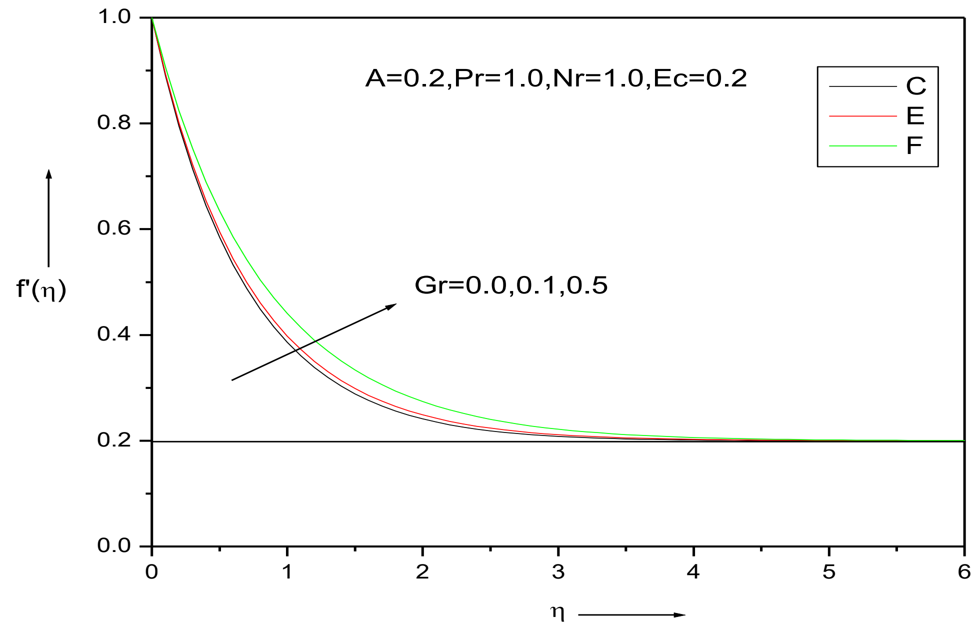

Fig.2.Velocity profile various values of Gr

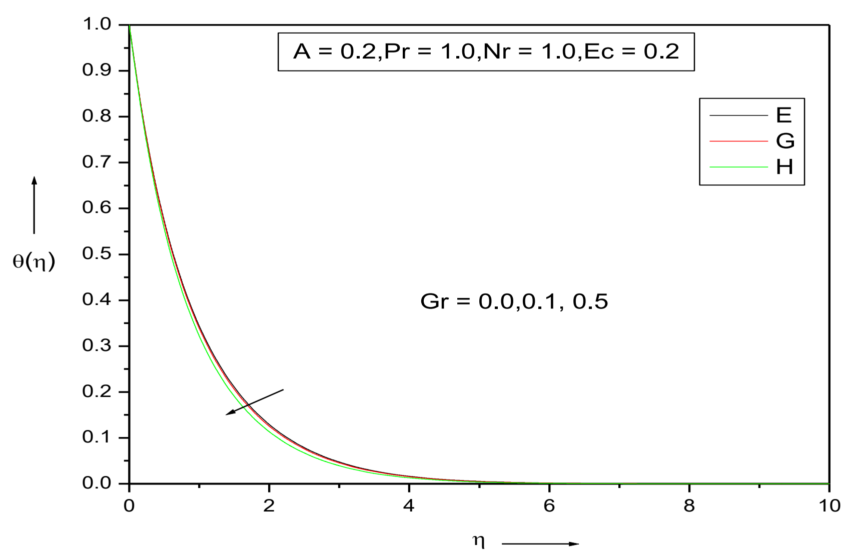

Fig.3.Temperature profile for various values of Gr

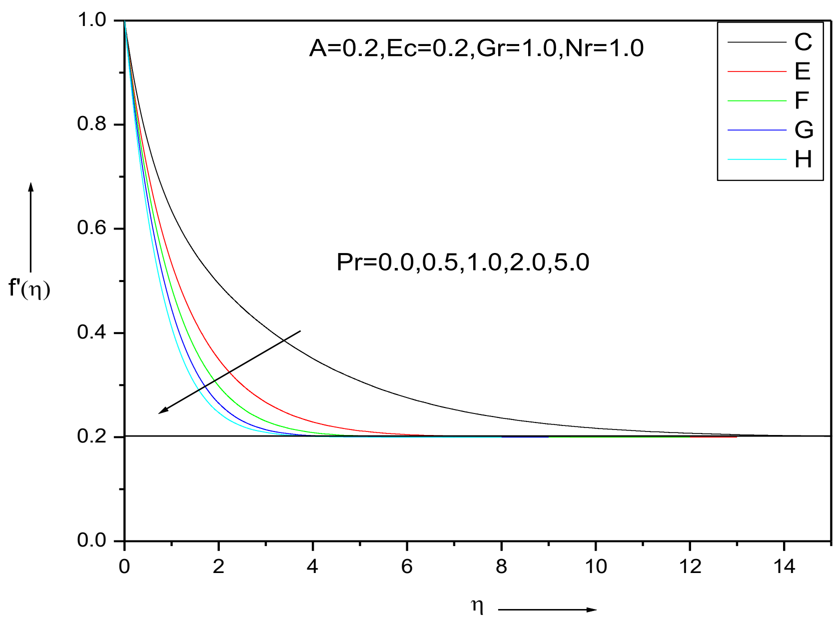

Fig.4.Velocity profile various values of Pr

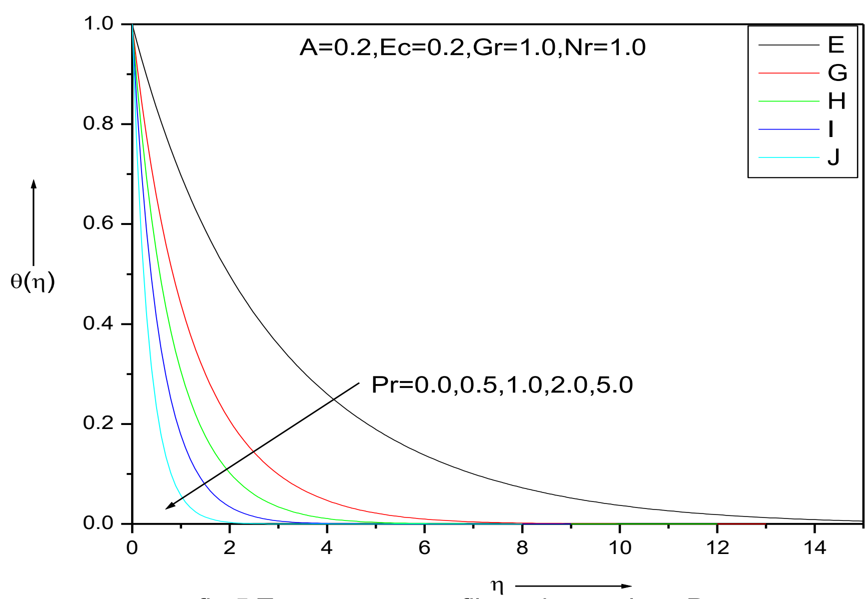

fig.5.Temperature profile various values Pr

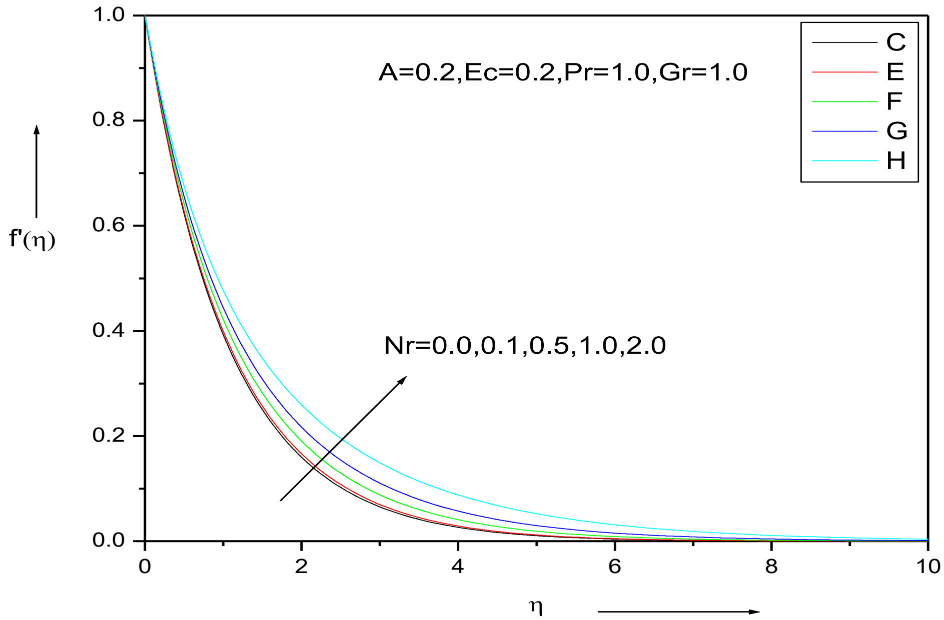

Fig.6.Velocity profile for various values of Nr

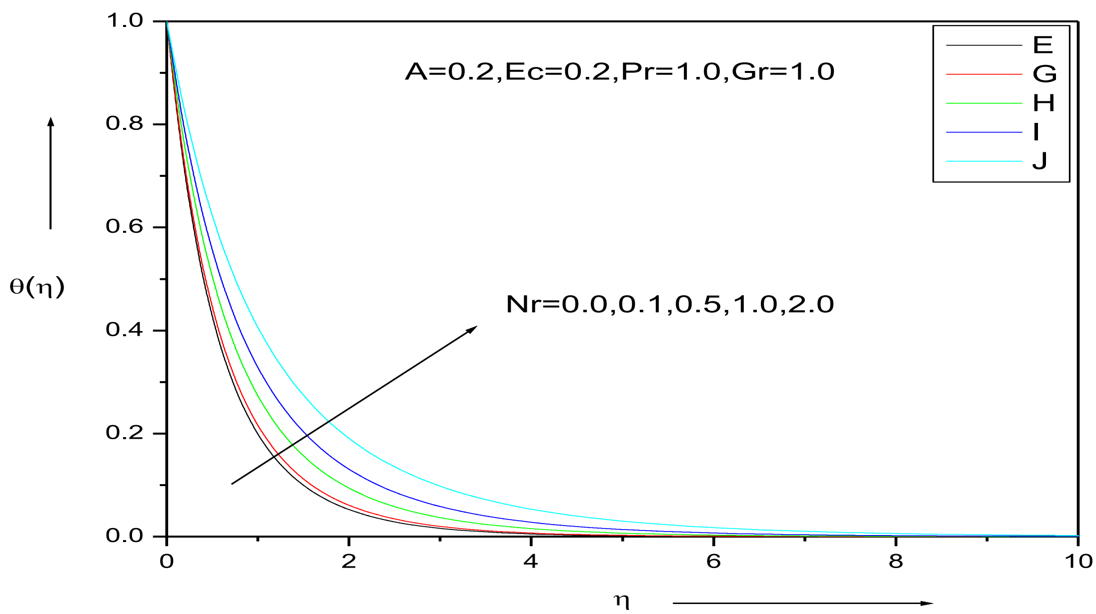

Fig.7.Temperature profile for various values of Nr

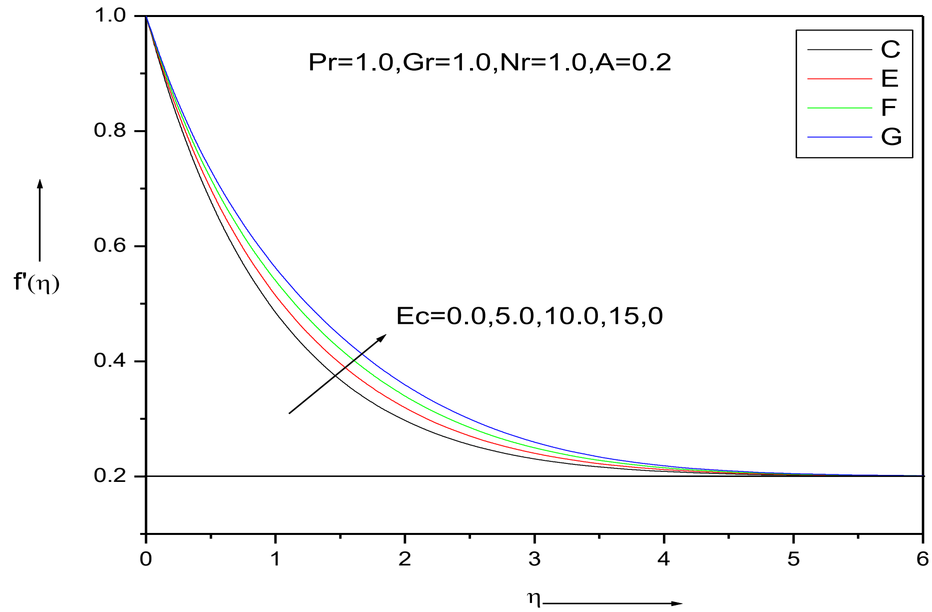

Fig.8.Velocity profile for various values of Ec

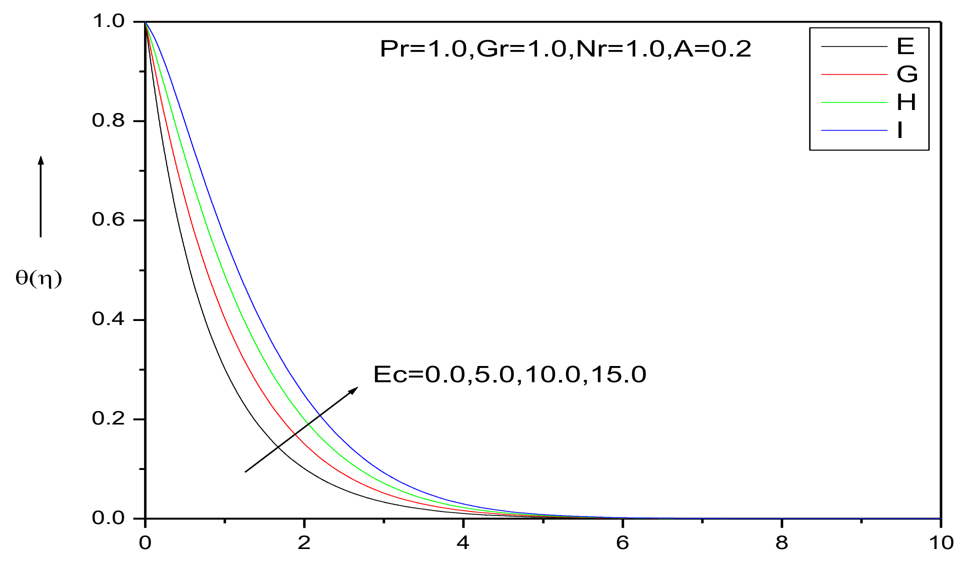

Fig.9.Temperature profile for various values of Ec

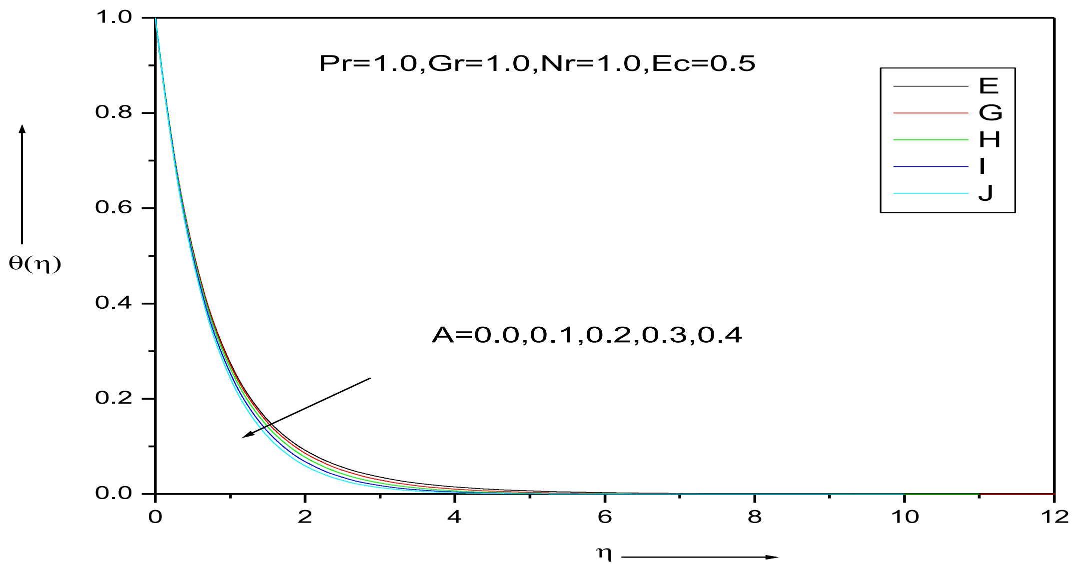

Fig.10.Temperature profile for various values of velocity ratio

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